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Couplings of vector-spinor representation for SO(10) model building

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Abstract: Higgs multiplet in the vector-spinor representations of $SO(10)$, i.e., the $144 + \overline{144}$ multiplet can break the $SO(10)$ gauge symmetry spontaneously in one step down to the Standard Model gauge group symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ and a recent analysis has used such vector-spinors for building a new class of $SO(10)$ grand unification models. Here we discuss the techniques for the computation of several classes of vector-spinor couplings using the recent result on the SO(2N) vertex expansion. The computations include the cubic couplings of the vector-spinors with $SO(10)$ tensors, quartic self-couplings of the vector-spinors, and couplings of the vector-spinors with spinor representations of $SO(10)$. The last set include couplings of vector-spinors with the 16-plets of quarks and lepton and with the 16 and $\overline{16}$ of Higgs. These couplings provide a tool for further development of the $SO(10)$ grand unification using vector-spinor representations. These include study of quark-lepton masses, analysis of dimension five operators including baryon and lepton number violating operators, and study of neutrino masses and mixings. Illustrative examples are given for their computation using a sample of vector-spinor couplings. The vector-spinor couplings arise in a wide class of models when one considers higher dimensional operators such as those that arise in the analysis of Planck scale physics and thus the techniques as well as the explicit couplings discussed here should find a wider application. .

Keywords: Supersymmetric Effective Theories, Spontaneous Symmetry Breaking, Supersymmetric gauge theory, Quark Masses and SM Parameters.

Contents

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1. Introduction

$SO(10)$ is a favored group for the unification of the electro-weak and the strong interactions [1, 2]. However, there is a wide array of possibilities for model building within the gauge group. Thus while the remarkable feature of $SO(10)$ is that it unifies one generation of quarks and leptons within one irreducible representation, i.e., the 16 plet representation, the Higgs sector of the theory is largely unconstrained and thus there exist a wide variety of models which differ by the choice of the Higgs sector of the theory. In most models the Higgs sector is generally quite elaborate involving several Higgs multiplets necessary for the breaking of $SO(10)$ symmetry in steps down to the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. An interesting recent proposal made by Babu, Gogoladze and the authors is to use a single pair of $144 + \overline{144}$ multiplet to break the $SO(10)$ gauge group in one step down to the Standard Model gauge symmetry [3]. The couplings involving the 144 and $\overline{144}$ are rather intricate and not easily computable. However, significant progress has occurred recently in how one may compute couplings involving spinor and tensor representations of SO(10) [5 − 7]. An important result in such constructions is the so called Basic Theorem deduced in ref. [5] using oscillator techniques [8, 9] which facilitates the computations of vertices involving spinor and tensor $SO(10)$ representations. Thus using the basic theorem, couplings of the type $16 \times 16 \times 10$, $16 \times 10 \times 120$, $16 \times 16 \times \overline{126}$ and $16^\dagger \times 16 \times 1$, $16^\dagger \times 16 \times 1$ were computed in ref. [5] and further applications of the technique were made in ref. [10]. Now the couplings of the 144 and $\overline{144}$ are more involved. This is so because of two factors: first we are dealing with a vector-spinor rather than just a spinor representation of $SO(10)$. Second the vector-spinor is constrained in order that it correspond to the irreducible 144 or $\overline{144}$ representation of $SO(10)$. Nonetheless, we will find that the techniques of ref. [5] appropriately adopted to this case will prove very useful in the analysis of $SO(10)$ vertices: cubic, quartic or of higher order. In this paper we will limit ourselves to the analysis of cubic and quartic interactions where the 144 and $\overline{144}$ are involved. The detailed knowledge of the couplings of a gauge group are useful in model building [11], and in extracting the implications of the models for spontaneous symmetry breaking, neutrino oscillations [12] proton decay [13 − 15], computation of the mass spectra and a variety of other applications. We note that the couplings involving the 144 -plet arise naturally in a wide class of SO(10) models. Thus, for example, in a conventional $SO(10)$ model with a 10 and a 45 of Higgs one would have a dimension five operator $(16 \times 10_H)_{\overline{144}}.(16 \times 45_H)_{144}$ where the subscript means that the interaction is being mediated by the representation in the subscript. The number of such operators is rather large but the techniques discussed here can be utilized for their computation . This provides the motivation for the analysis given in this paper.

The outline of the rest of the paper is as follows: In section 2 we give a brief summary of previous results which are essential for the developments of the succeeding sections. Here we discuss the generators of $SO(10)$ in the $SU(5) \times U(1)$ basis using the oscillator approach. We then state the so called Basic Theorem that significantly facilitates the computation of couplings for spinor and tensor representations in $SO(10)$. In section 3 we address the question of how one may treat the 144 irreducible representation through the use of a constrained vector-spinor. This is so because, the vector-spinor in $SO(10)$ has $16 \times 10 = 160$ components, and we need a constraint to eliminate sixteen components to get the irreducible 144-plet tensor. In this section we also decompose the 144 in representations of $SU(5) \times U(1)$ and define their normalizations. An analysis of the cubic couplings of 144 and $\overline{144}$ with the 1, 45, 210 tensor representations and with 10, 120 and $\overline{126}$ tensor representations is given in section 4. In section 5 we give some illustrative examples of how the vector-spinor couplings are to be used in model building. Here we show the spontaneous breaking of SO(10) to the Standard Model gauge group in a single step, carry out an analysis of Higgs doublet and Higgs triplet mass matrices and obtain the condition under which mass less doublets can be obtained. We show how Yukawa couplings and quark-lepton mass terms can arise after spontaneous breaking of $SO(10)$ and compute lepton and baryon number number violating dimension five operators which contribute to proton decay. Specifically we show that these operators now receive contributions from several sources raising the possibility of suppression of the proton decay by cancellation. Conclusions are given in section 6.

Further details of the analysis are given in several appendices. In appendix A, reduction of SO(10) fields in $SU(5)$ representations, and further reduction in $SU(3)_C \times SU(2)_L$ representations is discussed. Here we also give normalizations and define notation used in the rest of the paper. In appendix B we discuss the gauge couplings of the 144 and $\overline{144}$ with the singlet gauge field and with a 45 plet of gauge field belonging to the adjoint representation of $SO(10)$. In appendix C we compute the self couplings of the vector-spinor representations. These couplings cannot be cubic and the allowed couplings must at least be quartic or higher. These can be of several types. Thus $144 \times \overline{144}$ can couple with $144 \times \overline{144}$ by mediation by 1, 45 and 210. Additionally, there are couplings where 144×144 and $\overline{144} \times \overline{144}$ can couple with 144×144 and $\overline{144} \times \overline{144}$ either by mediation by 10, 120 or $126 + \overline{126}$. Thus there are a variety of quartic self-couplings involving spinors. In appendix D we discuss the couplings of vector-spinors with the 16-plet of matter. Here we consider couplings where 144×144 and $\overline{144} \times \overline{144}$ couple with 16×16 plets of quark-lepton matter multiplets via mediation by 10, 120 and $126 + \overline{126}$. Some further details of the quartic couplings from 10-plet mediation are given in appendix E, and similar details for 120-plet mediation are given in appendix F, and from $126 + \overline{126}$ are given in appendix G.

2. Preliminaries

An efficient decomposition of the $SO(10)$ vertices is in the $SU(5) \times U(1)$ basis. In this section we give the basic formulae for the decomposition of the $SO(10)$ generators in this basis and further we give the Basic Theorem for the computation of the SO(10) vertices. We begin by defining the Clifford elements, Γ_μ ($\mu = 1, 2, ..., 10$) in terms of creation and destruction operators, b_i and $b_i^\dagger$ ($i = 1, 2, ..., 5$) [8, 9]

$$\Gamma_{2i} = (b_i + b_i^\dagger); \quad \Gamma_{2i-1} = -i(b_i - b_i^\dagger) \tag{2.1}$$

so that

$$\{\Gamma_\mu, \Gamma_\nu\} = 2\delta_{\mu\nu}. \tag{2.2}$$

where

$$\{b_i, b_j^\dagger\} = \delta_i^j; \quad \{b_i, b_j\} = 0; \quad \{b_i^\dagger, b_j^\dagger\} = 0 \tag{2.3}$$

and that the $SU(5)$ singlet state $|0>$ satisfies $b_i|0>= 0$. The 45 generators of $SO(10)$ in the spinor representation are

$$\Sigma_{\rho\sigma} = \frac{1}{2i}[\Gamma_\rho, \Gamma_\sigma] \tag{2.4}$$

In the analysis of $SO(10)$ invariant interactions one also needs the equivalent of charge conjugation operator given by

$$B = \prod_{\mu=odd} \Gamma_\mu = -i \prod_{k=1}^{5} (b_k - b_k^\dagger) \tag{2.5}$$

The semi-spinors $\Psi_{(\pm)\acute{a}}$ ($\acute{a} = 1, 2, 3$) transforms as a $16(\overline{16})$-dimensional irreducible representation of $SO(10)$ and contains $1 + \overline{5} + 10(1 + 5 + \overline{10})$ in its $SU(5)$ decomposition. They are given by

$$|\Psi_{(+)\acute{a}}>= |0> \mathbf{M}_{\acute{a}} + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \mathbf{M}_{\acute{a}}^{ij} + \frac{1}{24} \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathbf{M}_{\acute{a}i} \tag{2.6}$$

$$|\Psi_{(-)\acute{a}}>= b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger |0> \mathbf{N}_{\acute{a}} + \frac{1}{12} \epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathbf{N}_{\acute{a}ij} + b_i^\dagger |0> \mathbf{N}_{\acute{a}}^i \tag{2.7}$$

We now review the recently developed technique [5] for the analysis of $SO(2N)$ invariant couplings which allows a full exhibition of the $SU(N)$ invariant content of the spinor and tensor representations. The technique utilizes a basis consisting of a specific set of reducible $SU(N)$ tensors in terms of which the $SO(2N)$ invariant couplings have a simple expansion. To that end, we note that the natural basis for the expansion of the $SO(2N)$ vertex is in terms of a specific set of $SU(N)$ reducible tensors, Φ_{c_k} and $\Phi_{\overline{c}_k}$ which we define

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as $A^k \equiv \Phi_{c_k} \equiv \Phi_{2k} + i\Phi_{2k-1}$, $A_k \equiv \Phi_{\bar{c}_k} \equiv \Phi_{2k} - i\Phi_{2k-1}$. This is extended immediately to define the quantity $\Phi_{c_i c_j \bar{c}_k ..}$ with an arbitrary number of unbarred and barred indices where each c index can be expanded out so that $A^i A^j A_k ... = \Phi_{c_i c_j \bar{c}_k ...} = \Phi_{2i c_j \bar{c}_k ...} + i\Phi_{2i-1 c_j \bar{c}_k ...}$ etc.. Thus, for example, the quantity $\Phi_{c_i c_j \bar{c}_k ... c_N}$ is a sum of 2^N terms gotten by expanding all the c indices. $\Phi_{c_i c_j \bar{c}_k ... c_n}$ is completely anti-symmetric in the interchange of its c indices whether unbarred or barred: $\Phi_{c_i \bar{c}_j c_k ... \bar{c}_n} = -\Phi_{c_k \bar{c}_j c_i ... \bar{c}_n}$. Further, $\Phi^*_{c_i \bar{c}_j c_k ... \bar{c}_n} = \Phi_{\bar{c}_i c_j \bar{c}_k ... c_n}$ etc.. We now make the observation [6] that the object $\Phi_{c_i c_j \bar{c}_k ... c_n}$ transforms like a reducible representation of $SU(N)$. Thus if we are able to compute the $SO(2N)$ invariant couplings in terms of these reducible tensors of $SU(N)$ then there remains only the further step of decomposing the reducible tensors into their irreducible parts. These results are codified in the so called The Basic Theorem which we discuss next.

The vertex $\Gamma_\mu \Gamma_\nu \Gamma_\lambda .. \Gamma_\sigma \ \Phi_{\mu\nu\lambda..\sigma}$ where $\Phi_{\mu\nu\lambda..\sigma}$ is a Higgs tensor, appears often in $SO(2N)$ invariant couplings and can be expanded in the following form

$$\Gamma_\mu \Gamma_\nu \Gamma_\lambda .. \Gamma_\sigma \Phi_{\mu\nu\lambda ... \sigma} = b_i^\dagger b_j^\dagger b_k^\dagger ... b_n^\dagger \Phi_{c_i c_j c_k ... c_n} + \left(b_i b_j^\dagger b_k^\dagger ... b_n^\dagger \Phi_{\bar{c}_i c_j c_k ... c_n} + \ perms \right) \\ + \left(b_i b_j b_k^\dagger ... b_n^\dagger \Phi_{\bar{c}_i \bar{c}_j c_k ... c_n} + \ perms \right) + ... + \left(b_i b_j b_k ... b_{n-1} b_n^\dagger \Phi_{\bar{c}_i \bar{c}_j \bar{c}_k ... \bar{c}_{n-1} c_n} + \ perms \right) \\ + b_i b_j b_k ... b_n \Phi_{\bar{c}_i \bar{c}_j \bar{c}_k ... \bar{c}_n} \quad (2.8)$$

As mentioned above, the object $\Phi_{c_i c_j \bar{c}_k ... c_n}$ transforms like a reducible representation of $SU(N)$ which can be further decomposed in its irreducible parts.

3. 144 and $\overline{144}$ as Constrained Vector-Spinor Mutiplets

In this section we discuss the $SU(5)$ particle content of the 144 and $\overline{144}$ vector-spinors and their expansion in terms of oscillator modes. We begin by discussing first the field content of the reducible vector-spinor 160 and $\overline{160}$ multiplets [3]:

$$|\Psi_{(+)\acute{a}\mu}>= |0> \mathbf{P}_{\acute{a}\mu} + \frac{1}{2} b_i^\dagger b_j^\dagger |0> \mathbf{P}^{ij}_{\acute{a}\mu} + \frac{1}{24} \epsilon^{ijklm} b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathbf{P}_{\acute{a}i\mu} \quad (3.1)$$

$$|\Psi_{(-)\acute{b}\mu}>= b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger |0> \mathbf{Q}_{\acute{b}\mu} + \frac{1}{12} \epsilon^{ijklm} b_k^\dagger b_l^\dagger b_m^\dagger |0> \mathbf{Q}_{\acute{b}ij\mu} + b_i^\dagger |0> \mathbf{Q}^i_{\acute{b}\mu} \quad (3.2)$$

where the lower case Latin letters $i, j, k, l, m, ... = 1, 2, ..., 5$ are $SU(5)$ indices, the lower case Greek letters $\mu, \nu, \rho, ... = 1, 2, ..., 10$ represent $SO(10)$ indices, while the lower case Latin letters with accent $\acute{a}, \acute{b}, \acute{c}, \acute{d} = 1, 2, 3$ are generation indices. The $SU(5)$ field content of $160 + \overline{160}$ multiplet is

$$\overline{160}(\Psi_{(+)\mu}) = 1(\widehat{\mathbf{P}}) + \bar{5}(\mathbf{P}_i) + 5(\mathbf{P}^i) + 5(\widehat{\mathbf{P}}^i) + \overline{10}(\mathbf{P}_{ij}) + \overline{10}(\widehat{\mathbf{P}}_{ij}) + \overline{15}(\mathbf{P}^{(S)}_{ij}) \\ + 24(\mathbf{P}^i_j) + \overline{40}(\mathbf{P}^i_{jkl}) + 45(\mathbf{P}^{ij}_k) \quad (3.3)$$

$$160(\Psi_{(-)\mu}) = 1(\widehat{\mathbf{Q}}) + 5(\mathbf{Q}^i) + \bar{5}(\mathbf{Q}_i) + \bar{5}(\widehat{\mathbf{Q}}_i) + 10(\mathbf{Q}^{ij}) + 10(\widehat{\mathbf{Q}}^{ij}) + 15(\mathbf{Q}^{ij}_{(S)}) \\ + 24(\mathbf{Q}^i_j) + 40(\mathbf{Q}^{ijk}_l) + \overline{45}(\mathbf{Q}^i_{jk}) . \quad (3.4)$$

Details of the decomposition are given in appendix A.

The vector-spinor $|\Psi_{(+)\mu}>$ is unconstrained, has 160 components and is reducible. To see how the 160 plet can be reduced, we note that $\Gamma_\mu|\Psi_{(+)\mu}>$ is a 16 dimensional $SO(10)$ spinor. Thus one way to define an irreducible 144 ($\overline{144}$) dimensional vector-spinor is to impose the constraint

$$\Gamma_\mu|\Upsilon_{(\pm)\mu}>=0 \tag{3.5}$$

We explore now the implication of the above constraint. The contraction of Γ_μ with the 160+$\overline{160}$ multiplet $|\Psi_{(\pm)\mu}>$ gives

$$\begin{aligned}
\Gamma_\mu|\Psi_{(+)\mu}>=b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger|0>\widehat{\mathbf{P}}+\frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0>\Big(\mathbf{P}_{ij}+6\widehat{\mathbf{P}}_{ij}\Big)\\
+b_i^\dagger|0>\Big(\mathbf{P}^i+\widehat{\mathbf{P}}^i\Big)\\
\Gamma_\mu|\Psi_{(-)\mu}>=|0>\widehat{\mathbf{P}}+\frac{1}{2}b_i^\dagger b_j^\dagger|0>\Big(\mathbf{Q}^{ij}+6\widehat{\mathbf{Q}}^{ij}\Big)\\
+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\Big(\mathbf{Q}_i+\widehat{\mathbf{Q}}_i\Big)
\end{aligned} \tag{3.6}$$

Thus to get the 144 and $\overline{144}$ spinor, $|\Upsilon_{(\pm)\mu}>$, we need to impose the following conditions:

$$\begin{aligned}
\widehat{\mathbf{P}}=0,\quad \widehat{\mathbf{P}}^i=-\mathbf{P}^i,\quad \widehat{\mathbf{P}}_{ij}=-\frac{1}{6}\mathbf{P}_{ij}\\
\widehat{\mathbf{Q}}=0,\quad \widehat{\mathbf{Q}}_i=-\mathbf{Q}_i,\quad \widehat{\mathbf{Q}}^{ij}=-\frac{1}{6}\mathbf{Q}^{ij}
\end{aligned} \tag{3.7}$$

Hence, we have following relation

$$|\Upsilon_{(\pm)\mu}>=\big(|\Psi_{(\pm)\mu}>\big)_{constraint\ of\ Eq.(3.7)} \tag{3.8}$$

The above implies that certain components of the 160 and $\overline{160}$ multiplets are either zero or are related thus reducing the number of independent components from 160 to 144. For completeness, we give the expansion of the constrained 144 and $\overline{144}$ vector-spinors in its oscillator modes

$$\begin{aligned}
\binom{\overline{144}}{144}:\quad |\Upsilon_{(\pm)\mu}>=\big(|\Upsilon_{(\pm)c_n}>,|\Upsilon_{(\pm)\bar{c}_n}>\big)\\
|\Upsilon_{(+)c_n}>=|0>\mathbf{P}^n+\frac{1}{2}b_i^\dagger b_j^\dagger|0>\left[\epsilon^{ijklm}\mathbf{P}^n_{klm}-\frac{1}{6}\epsilon^{ijnlm}\mathbf{P}_{lm}\right]\\
+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\mathbf{P}^n_i\\
|\Upsilon_{(+)\bar{c}_n}>=|0>\mathbf{P}_n+\frac{1}{2}b_i^\dagger b_j^\dagger|0>\left[\mathbf{P}^{ij}_n+\frac{1}{4}\left(\delta^i_n\mathbf{P}^j-\delta^j_n\mathbf{P}^i\right)\right]\\
+\frac{1}{24}\epsilon^{ijklm}b_j^\dagger b_k^\dagger b_l^\dagger b_m^\dagger|0>\left[\frac{1}{2}\mathbf{P}_{in}+\frac{1}{2}\mathbf{P}^{(S)}_{in}\right]\\
|\Upsilon_{(-)c_n}>=b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger|0>\mathbf{Q}^n+\frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0>\left[\mathbf{Q}^n_{ij}+\frac{1}{4}\left(\delta^n_i\mathbf{Q}_j-\delta^n_j\mathbf{Q}_i\right)\right]\\
+b_i^\dagger|0>\left[\frac{1}{2}\mathbf{Q}^{in}+\frac{1}{2}\mathbf{Q}^{in}_{(S)}\right]
\end{aligned}$$

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$$|\Upsilon_{(-)\bar{c}_n}>=b_1^\dagger b_2^\dagger b_3^\dagger b_4^\dagger b_5^\dagger|0>\mathbf{Q}_n+\frac{1}{12}\epsilon^{ijklm}b_k^\dagger b_l^\dagger b_m^\dagger|0>\left[\epsilon_{ijklm}\mathbf{Q}_n^{klm}-\frac{1}{6}\epsilon_{ijnlm}\mathbf{Q}^{lm}\right] +b_i^\dagger|0>\mathbf{Q}_n^i\,. \tag{3.9}$$

4. Higgs sector cubic couplings

In this section we compute the cubic couplings in the superpotential involving two vector-spinors and one each of the tensors 1, 10, 45, 120, 210, and $\overline{126}$ plet of Higgs. We discuss their $SU(5)\times U(1)$ decomposed form below.

4.1 The $(\mathbf{144}\times\overline{\mathbf{144}}\times\mathbf{1})$ couplings

The $(144\times\overline{144}\times 1)$ coupling structure in the superpotential is

$$\mathsf{W}^{(1)}=h^{(1)}_{\acute{a}\acute{b}}<\Upsilon^*_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu}>\Phi \tag{4.1}$$

Where Φ is the 1-plet of Higgs field. A computation of this coupling using the techniques described in sections 2 and 3 gives the following result in the $SU(5)\times U(1)$ decomposed form

$$\mathsf{W}^{(1)}=ih^{(1)}_{\acute{a}\acute{b}}\left[\frac{3}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^i_{\acute{b}}+\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}+\frac{1}{10}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}+\frac{1}{2}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij}\right.$$
$$\left.+\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^j_{\acute{b}i}-\frac{1}{6}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^l_{\acute{b}ijk}-\frac{1}{2}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\right]\mathsf{H}\,. \tag{4.2}$$

4.2 The $(\mathbf{144}\times\overline{\mathbf{144}}\times\mathbf{45})$ couplings

The $(144\times\overline{144}\times 45)$ couplings in the superpotential is

$$\mathsf{W}^{(45)}=\frac{1}{2!}h^{(45)}_{\acute{a}\acute{b}}<\Upsilon^*_{(-)\acute{a}\mu}|B\Sigma_{\rho\sigma}|\Upsilon_{(+)\acute{b}\mu}>\Phi_{\rho\sigma} \tag{4.3}$$

where $\Phi_{\rho\sigma}$ represents the 45-plet of Higgs field. A computation of this coupling using the techniques of sections 2 and 3 gives the following result in the $SU(5)\times U(1)$ decomposed form

$$\mathsf{W}^{(45)}=h^{(45)}_{\acute{a}\acute{b}}\left\{\left[\frac{3}{\sqrt{10}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^j_{\acute{b}i}+\frac{11}{10\sqrt{10}}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}+\frac{3}{\sqrt{10}}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij}+\frac{1}{2\sqrt{10}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\right.\right.$$
$$\left.-\frac{19}{5\sqrt{10}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^i_{\acute{b}}-\sqrt{\frac{5}{2}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}+\frac{1}{6\sqrt{10}}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^l_{\acute{b}ijk}\right]\mathsf{H}$$
$$+\left[-\frac{1}{\sqrt{2}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{lm}_{\acute{b}k}-\frac{1}{\sqrt{10}}\mathcal{Q}^{l\mathbf{T}}_{\acute{a}}\mathcal{P}^m_{\acute{b}}+\frac{2}{\sqrt{15}}\mathcal{Q}^{lk\mathbf{T}}_{\acute{a}}\mathcal{P}^m_{\acute{b}k}+\frac{1}{\sqrt{2}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}n}\mathcal{P}^n_{\acute{b}k}\right.$$
$$+\frac{7}{20\sqrt{3}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}jk}-\frac{1}{3\sqrt{10}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}n}\mathcal{P}^n_{\acute{b}ijk}-\frac{1}{4}\sqrt{\frac{3}{5}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}ij}\mathcal{P}_{\acute{b}nk}$$
$$\left.+\frac{1}{4}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{(S)}_{\acute{b}nk}\right]\mathsf{H}_{lm}$$

$$
\begin{aligned}
&+\Bigg[-\frac{1}{\sqrt{2}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}lm}\mathcal{P}_{\acute{b}k}+\frac{1}{\sqrt{10}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}l}\mathcal{P}_{\acute{b}m}+\frac{2}{\sqrt{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}l}\mathcal{P}_{\acute{b}km}+\frac{1}{\sqrt{2}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}klm}\\
&+\frac{7}{20\sqrt{3}}\epsilon_{ijklm}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}^{k}_{\acute{b}}-\frac{1}{3\sqrt{10}}\epsilon_{ijklm}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}}+\frac{1}{4}\sqrt{\frac{3}{5}}\epsilon_{ijklm}\mathcal{Q}^{in\mathbf{T}}_{\acute{a}}\mathcal{P}^{jk}_{\acute{b}n}\\
&+\frac{1}{4}\epsilon_{ijklm}\mathcal{Q}^{in\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{jk}_{\acute{b}n}\Bigg]\mathsf{H}^{lm}\\
&+\Bigg[\sqrt{2}\mathcal{Q}^{l\mathbf{T}}_{\acute{a}ik}\mathcal{P}^{kj}_{\acute{b}l}-\frac{1}{\sqrt{10}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}ik}\mathcal{P}^{k}_{\acute{b}}+\frac{1}{\sqrt{10}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}^{kj}_{\acute{b}i}-\frac{3}{10\sqrt{2}}\mathcal{Q}^{\mathbf{T}}_{ai}\mathcal{P}^{j}_{\acute{b}}\\
&+\frac{1}{\sqrt{2}}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}m}\mathcal{P}^{m}_{\acute{b}kli}-\frac{1}{\sqrt{15}}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}kl}-\frac{1}{\sqrt{15}}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}ikl}+\frac{1}{15\sqrt{2}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ki}\\
&-\sqrt{\frac{3}{10}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ki}+\sqrt{\frac{3}{10}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{a}}\mathcal{P}_{\acute{b}ki}-\frac{1}{\sqrt{2}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ki}-\sqrt{2}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}i}\Bigg]\mathsf{H}^{i}_{j}\Bigg\}.
\end{aligned}
\tag{4.4}
$$

4.3 The $(\mathbf{144}\times\overline{\mathbf{144}}\times\mathbf{210})$ couplings

The $(144\times\overline{144}\times 210)$ coupling structure is

$$
\mathsf{W}^{(210)}=\frac{1}{4!}h^{(210)}_{\acute{a}\acute{b}}<\Upsilon^{*}_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda]}|\Upsilon_{(+)\acute{b}\mu}>\Phi_{\nu\rho\sigma\lambda}
\tag{4.5}
$$

where $\Phi_{\nu\rho\sigma\lambda}$ represents the 210-plet of Higgs field. A computation of the couplings using the techniques of sections 2 and 3 gives the following result in the $SU(5)\times U(1)$ decomposed form

$$
\begin{aligned}
\mathsf{W}^{(210)}=ih^{(210)}_{\acute{a}\acute{b}}\Bigg\{&\Bigg[\frac{1}{2\sqrt{15}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}+\frac{1}{4\sqrt{15}}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}+\frac{1}{\sqrt{15}}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij}+\frac{1}{4\sqrt{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\\
&+\frac{7}{10}\sqrt{\frac{3}{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}+\frac{1}{2}\sqrt{\frac{5}{3}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}+\frac{1}{12\sqrt{15}}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}ijk}\Bigg]\mathsf{H}\\
&+\Bigg[-\frac{1}{2\sqrt{2}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{lm}_{\acute{b}k}-\frac{1}{2\sqrt{10}}\mathcal{Q}^{l\mathbf{T}}_{\acute{a}}\mathcal{P}^{m}_{\acute{b}}+\frac{1}{3\sqrt{15}}\mathcal{Q}^{lk\mathbf{T}}_{\acute{a}}\mathcal{P}^{m}_{\acute{b}k}-\frac{1}{6\sqrt{2}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}k}\\
&+\frac{1}{4}\sqrt{\frac{3}{10}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}jk}-\frac{1}{6\sqrt{10}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}ijk}+\frac{1}{8\sqrt{15}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}ij}\mathcal{P}_{\acute{b}nk}\\
&-\frac{1}{24}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{(S)}_{\acute{b}nk}\Bigg]\mathsf{H}_{lm}\\
&+\Bigg[-\frac{1}{2\sqrt{2}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}lm}\mathcal{P}_{\acute{b}k}-\frac{1}{2\sqrt{10}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}l}\mathcal{P}_{\acute{b}m}+\frac{1}{3\sqrt{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}l}\mathcal{P}_{\acute{b}km}+\frac{1}{6\sqrt{2}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}klm}\\
&-\frac{1}{4}\sqrt{\frac{3}{10}}\epsilon_{ijklm}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}^{k}_{\acute{b}}+\frac{1}{6\sqrt{10}}\epsilon_{ijklm}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}}+\frac{1}{8\sqrt{15}}\epsilon_{ijklm}\mathcal{Q}^{in\mathbf{T}}_{\acute{a}}\mathcal{P}^{jk}_{\acute{b}n}\\
&+\frac{1}{24}\epsilon_{ijklm}\mathcal{Q}^{in\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{jk}_{\acute{b}n}\Bigg]\mathsf{H}^{lm}\\
&+\Bigg[-\frac{1}{3\sqrt{2}}\mathcal{Q}^{l\mathbf{T}}_{\acute{a}ik}\mathcal{P}^{kj}_{\acute{b}l}+\frac{1}{6\sqrt{10}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}ik}\mathcal{P}^{k}_{\acute{b}}-\frac{1}{6\sqrt{10}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}^{kj}_{\acute{b}i}+\frac{1}{20\sqrt{2}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{j}_{\acute{b}}\\
&-\frac{1}{6\sqrt{2}}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}m}\mathcal{P}^{m}_{\acute{b}ikl}+\frac{1}{6\sqrt{15}}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}i}\mathcal{P}^{kl}_{\acute{b}}+\frac{1}{6\sqrt{15}}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}ikl}+\frac{5\sqrt{2}}{9}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ki}
\end{aligned}
$$

$$
-\frac{1}{2}\sqrt{\frac{3}{10}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ki}+\frac{1}{2}\sqrt{\frac{3}{10}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{a}}\mathcal{P}_{\acute{b}ki}-\frac{1}{2\sqrt{2}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ki}-\frac{1}{\sqrt{2}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}i}\Bigg]\mathsf{H}^{i}_{j}
$$

$$
+\left[-\frac{1}{\sqrt{5}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ji}+\frac{1}{\sqrt{3}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ji}+2\sqrt{\frac{2}{15}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}\right]\mathsf{H}^{i}
$$

$$
+\left[\frac{1}{\sqrt{5}}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}j}+\frac{1}{\sqrt{3}}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}_{\acute{b}j}+2\sqrt{\frac{2}{15}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}}\right]\mathsf{H}_{i}
$$

$$
+\Bigg[\frac{1}{2\sqrt{6}}\mathcal{Q}^{m\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{kl}_{\acute{b}m}+\frac{1}{2\sqrt{30}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{l}_{\acute{b}}-\frac{1}{2\sqrt{30}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{kl}_{\acute{b}j}+\frac{1}{2\sqrt{6}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}mij}
$$

$$
+\frac{1}{3\sqrt{5}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}mj}+\frac{1}{3\sqrt{5}}\mathcal{Q}^{lm\mathbf{T}}_{\acute{a}}\mathcal{P}^{k}_{\acute{b}mij}+\frac{1}{15\sqrt{6}}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\Bigg]\mathsf{H}^{ij}_{kl}
$$

$$
+\Bigg[\frac{1}{6\sqrt{5}}\epsilon_{ijklm}\mathcal{Q}^{ip\mathbf{T}}_{\acute{a}}\mathcal{P}^{p}_{\acute{b}jn}+\frac{1}{6\sqrt{3}}\epsilon_{ijklm}\mathcal{Q}^{ip\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{jn}_{\acute{b}p}+\frac{1}{60}\epsilon_{ijklm}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}^{n}_{\acute{b}}
$$

$$
-\frac{1}{60}\epsilon_{ijklm}\mathcal{Q}^{in\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}}-\frac{1}{12\sqrt{15}}\epsilon_{ijklm}\mathcal{Q}^{in\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{j}_{\acute{b}}-\frac{1}{3\sqrt{6}}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}p}\mathcal{P}^{p}_{\acute{b}klm}+\frac{1}{3\sqrt{5}}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}k}\mathcal{P}_{\acute{b}lm}\Bigg]\mathsf{H}^{klm}_{n}
$$

$$
+\Bigg[\frac{1}{6\sqrt{5}}\epsilon^{ijklm}\mathcal{Q}^{p\mathbf{T}}_{\acute{a}in}\mathcal{P}_{\acute{b}jp}+\frac{1}{6\sqrt{3}}\epsilon^{ijklm}\mathcal{Q}^{p\mathbf{T}}_{\acute{a}in}\mathcal{P}^{(S)}_{\acute{b}jp}-\frac{1}{60}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}n}\mathcal{P}_{\acute{b}ij}
$$

$$
-\frac{1}{60}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}jn}-\frac{1}{12\sqrt{15}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{(S)}_{\acute{b}jn}+\frac{1}{3\sqrt{6}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}p}\mathcal{P}^{p}_{\acute{b}n}-\frac{1}{3\sqrt{5}}\mathcal{Q}^{lm\mathbf{T}}_{\acute{a}}\mathcal{P}^{k}_{\acute{b}n}\Bigg]\mathsf{H}^{n}_{klm}\Bigg\}. \tag{4.6}
$$

4.4 The $(\overline{\mathbf{144}}\times\overline{\mathbf{144}}\times\mathbf{10})$ couplings

The $(\overline{144}\times\overline{144}\times 10)$ couplings in the superpotential are given by

$$
\mathsf{W}^{(10)}=h^{(10)}_{\acute{a}\acute{b}}<\Upsilon^{*}_{(+)\acute{a}\mu}|B\Gamma_{\nu}|\Upsilon_{(+)\acute{b}\mu}>\Phi_{\nu} \tag{4.7}
$$

where Φ_ν represents the 10-plet of Higgs field. A computation of the couplings using the techniques of sections 2 and 3 gives the following result in the $SU(5)\times U(1)$ decomposed form

$$
\mathsf{W}^{(10)}=ih^{(10)(+)}_{\acute{a}\acute{b}}\Bigg\{\Bigg[\frac{1}{\sqrt{15}}\epsilon^{ijlmn}\mathcal{P}^{k\mathbf{T}}_{\acute{a}lmn}\mathcal{P}_{\acute{b}ik}+\frac{1}{3}\epsilon^{ijlmn}\mathcal{P}^{k\mathbf{T}}_{\acute{a}lmn}\mathcal{P}^{(S)}_{\acute{b}ik}-\frac{\sqrt{2}}{5}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{a}lm}\mathcal{P}_{\acute{b}ik}
$$

$$
+2\sqrt{2}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}i}-\sqrt{\frac{2}{5}}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}i}\Bigg]\mathsf{H}_{j}
$$

$$
+\Bigg[\frac{2\sqrt{3}}{5}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}jk}-\frac{4}{\sqrt{5}}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}jk}-2\sqrt{2}\mathcal{P}^{\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}k}+\sqrt{2}\mathcal{P}^{l\mathbf{T}}_{\acute{a}ijk}\mathcal{P}^{ij}_{\acute{b}l}
$$

$$
-\frac{2}{\sqrt{15}}\mathcal{P}^{\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\Bigg]\mathsf{H}^{k}\Bigg\} \tag{4.8}
$$

Here and in the rest of the paper we define

$$
h^{(i)(\pm)}_{\acute{a}\acute{b}}=\frac{1}{2}\left(h^{(i)}_{\acute{a}\acute{b}}\pm h^{(i)}_{\acute{b}\acute{a}}\right) \tag{4.9}
$$

where (i) is the specific tensor representation.

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4.5 The $(\overline{\mathbf{144}} \times \overline{\mathbf{144}} \times \mathbf{120})$ coupling

The $(\overline{144} \times \overline{144} \times 120)$ couplings in the superpotential are given by

$$\mathsf{W}^{(120)} = \frac{1}{3!} h^{(120)}_{\acute{a}\acute{b}} < \Upsilon^{*}_{(+)\acute{a}\mu} | B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\lambda]} | \Upsilon_{(+)\acute{b}\mu} > \Phi_{\nu\rho\lambda} \tag{4.10}$$

where $\Phi_{\nu\rho\lambda}$ represents the 120-plet of Higgs field. A computation of the couplings using the techniques of sections 2 and 3 gives the following result in the $SU(5) \times U(1)$ decomposed form

$$\begin{aligned}
\mathsf{W}^{(120)} = ih^{(120)(-)}_{\acute{a}\acute{b}} \Bigg\{ & \Bigg[-\frac{1}{3\sqrt{10}} \epsilon^{ijlmn} \mathcal{P}^{k\mathbf{T}}_{\acute{a}lmn} \mathcal{P}_{\acute{b}ik} - \frac{1}{3\sqrt{6}} \epsilon^{ijlmn} \mathcal{P}^{k\mathbf{T}}_{\acute{a}lmn} \mathcal{P}^{(S)}_{\acute{b}ik} + \frac{1}{5\sqrt{3}} \epsilon^{ijklm} \mathcal{P}^{\mathbf{T}}_{\acute{a}lm} \mathcal{P}_{\acute{b}ik} \\
& - \frac{2}{\sqrt{3}} \mathcal{P}^{ij\mathbf{T}}_{\acute{a}k} \mathcal{P}^{k}_{\acute{b}i} + \frac{1}{\sqrt{15}} \mathcal{P}^{i\mathbf{T}}_{\acute{a}} \mathcal{P}^{j}_{\acute{b}i} \Bigg] \mathsf{H}_j \\
& + \left[\frac{4}{\sqrt{15}} \mathcal{P}^{j\mathbf{T}}_{\acute{a}} \mathcal{P}_{\acute{b}jk} - 2\sqrt{\frac{2}{3}} \mathcal{P}^{j\mathbf{T}}_{\acute{a}} \mathcal{P}^{(S)}_{\acute{b}jk} - \frac{4}{\sqrt{3}} \mathcal{P}^{\mathbf{T}}_{\acute{a}j} \mathcal{P}^{j}_{\acute{b}k} \right] \mathsf{H}^k \\
& + \left[-\frac{1}{3\sqrt{3}} \epsilon^{ijlmn} \mathcal{P}^{\mathbf{T}}_{\acute{a}k} \mathcal{P}^{k}_{\acute{b}lmn} + \frac{1}{3}\sqrt{\frac{2}{5}} \epsilon^{ijklm} \mathcal{P}^{\mathbf{T}}_{\acute{a}k} \mathcal{P}_{\acute{b}lm} - \frac{4}{\sqrt{15}} \mathcal{P}^{k\mathbf{T}}_{\acute{a}} \mathcal{P}^{ij}_{\acute{b}k} - 2\sqrt{\frac{2}{5}} \mathcal{P}^{i\mathbf{T}}_{\acute{a}} \mathcal{P}^{j}_{\acute{b}} \right] \mathsf{H}_{ij} \\
& + \left[\frac{4\sqrt{2}}{5} \mathcal{P}^{k\mathbf{T}}_{\acute{a}i} \mathcal{P}_{\acute{b}jk} + 4\sqrt{\frac{2}{15}} \mathcal{P}^{k\mathbf{T}}_{\acute{a}i} \mathcal{P}^{(S)}_{\acute{b}jk} \right] \mathsf{H}^{ij} \\
& + \Bigg[-\frac{1}{3\sqrt{10}} \epsilon^{ijmnp} \mathcal{P}^{l\mathbf{T}}_{\acute{a}mnp} \mathcal{P}_{\acute{b}kl} - \frac{1}{3\sqrt{6}} \epsilon^{ijmnp} \mathcal{P}^{l\mathbf{T}}_{\acute{a}mnp} \mathcal{P}^{(S)}_{\acute{b}kl} + \frac{1}{5\sqrt{3}} \epsilon^{ijlmn} \mathcal{P}^{\mathbf{T}}_{\acute{a}mn} \mathcal{P}_{\acute{b}kl} \\
& + \frac{1}{3\sqrt{5}} \epsilon^{ijlmn} \mathcal{P}^{\mathbf{T}}_{\acute{a}mn} \mathcal{P}^{(S)}_{\acute{b}kl} - \frac{2}{\sqrt{3}} \mathcal{P}^{ij\mathbf{T}}_{\acute{a}l} \mathcal{P}^{l}_{\acute{b}k} - \frac{2}{\sqrt{15}} \mathcal{P}^{i\mathbf{T}}_{\acute{a}} \mathcal{P}^{j}_{\acute{b}k} \Bigg] \mathsf{H}^{k}_{ij} \\
& + \left[-\frac{2}{\sqrt{3}} \mathcal{P}^{j\mathbf{T}}_{\acute{a}klm} \mathcal{P}^{mn}_{\acute{b}j} - \frac{1}{\sqrt{15}} \mathcal{P}^{j\mathbf{T}}_{\acute{a}} \mathcal{P}^{n}_{\acute{b}jkl} - \frac{4}{3}\sqrt{\frac{2}{5}} \mathcal{P}^{\mathbf{T}}_{\acute{a}km} \mathcal{P}^{mn}_{\acute{b}l} + \frac{2\sqrt{2}}{15} \mathcal{P}^{\mathbf{T}}_{\acute{a}kl} \mathcal{P}^{n}_{\acute{b}} \right] \mathsf{H}^{kl}_{n} \Bigg\}
\end{aligned} \tag{4.11}$$

4.6 The $(\overline{\mathbf{144}} \times \overline{\mathbf{144}} \times \overline{\mathbf{126}})$ couplings

The $(\overline{144} \times \overline{144} \times \overline{126})$ coupling in the superpotential is

$$\mathsf{W}^{(\overline{126})} = \frac{1}{5!} h^{(\overline{126})}_{\acute{a}\acute{b}} < \Upsilon^{*}_{(+)\acute{a}\mu} | B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\theta]} | \Upsilon_{(+)\acute{b}\mu} > \overline{\Phi}_{\nu\rho\sigma\lambda\theta} \tag{4.12}$$

where $\overline{\Phi}_{\nu\rho\sigma\lambda\theta}$ represents the $\overline{126}$-plet of Higgs field. A computation of the couplings using the techniques of sections 2 and 3 gives the following result in the $SU(5) \times U(1)$ decomposed form

$$\begin{aligned}
\mathsf{W}^{(\overline{126})} = ih^{(\overline{126})(+)}_{\acute{a}\acute{b}} \Bigg\{ & \left[-\frac{8}{5\sqrt{3}} \mathcal{P}^{i\mathbf{T}}_{\acute{a}} \mathcal{P}_{\acute{b}i} \right] \mathsf{H} \\
& + \Bigg[\frac{\sqrt{2}}{5} \mathcal{P}^{j\mathbf{T}}_{\acute{a}} \mathcal{P}_{\acute{b}jk} - \frac{1}{\sqrt{5}} \mathcal{P}^{j\mathbf{T}}_{\acute{a}} \mathcal{P}^{(S)}_{\acute{b}jk} - \sqrt{\frac{2}{5}} \mathcal{P}^{\mathbf{T}}_{\acute{a}j} \mathcal{P}^{j}_{\acute{b}k} \\
& - \frac{1}{3\sqrt{10}} \mathcal{P}^{l\mathbf{T}}_{\acute{a}ijk} \mathcal{P}^{ij}_{\acute{b}l} + \frac{1}{15\sqrt{3}} \mathcal{P}^{\mathbf{T}}_{\acute{a}ij} \mathcal{P}^{ij}_{\acute{b}k} + \frac{1}{5\sqrt{15}} \mathcal{P}^{\mathbf{T}}_{\acute{a}ik} \mathcal{P}^{i}_{\acute{b}} \Bigg] \mathsf{H}^k
\end{aligned}$$

$$+\left[\frac{1}{3\sqrt{30}}\epsilon^{ijlmn}\mathcal{P}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}lmn}-\frac{1}{15}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}_{\acute{b}lm}\right.$$
$$\left.+\frac{2}{5}\sqrt{\frac{2}{3}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{ij}_{\acute{b}k}+\frac{2}{5}\sqrt{\frac{2}{15}}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}}\right]\mathsf{H}_{ij}$$
$$+\left[-\frac{2}{5}\mathcal{P}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}jk}-\frac{2}{\sqrt{15}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}^{(S)}_{\acute{b}jk}\right]\mathsf{H}^{ij}_{(S)}$$
$$+\left[\frac{1}{30}\epsilon^{ijmnp}\mathcal{P}^{l\mathbf{T}}_{\acute{a}mnp}\mathcal{P}_{\acute{b}kl}+\frac{1}{6\sqrt{15}}\epsilon^{ijmnp}\mathcal{P}^{l\mathbf{T}}_{\acute{a}mnp}\mathcal{P}^{(S)}_{\acute{b}kl}-\frac{1}{5\sqrt{30}}\epsilon^{ijlmn}\mathcal{P}^{\mathbf{T}}_{\acute{a}mn}\mathcal{P}_{\acute{b}kl}\right.$$
$$\left.-\frac{1}{15\sqrt{2}}\epsilon^{ijlmn}\mathcal{P}^{\mathbf{T}}_{\acute{a}mn}\mathcal{P}^{(S)}_{\acute{b}kl}+\sqrt{\frac{2}{15}}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}k}+\frac{1}{5}\sqrt{\frac{2}{3}}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}k}\right]\mathsf{H}^{k}_{ij}$$
$$\left.+\left[-\frac{1}{3\sqrt{15}}\mathcal{P}^{n\mathbf{T}}_{\acute{a}ijk}\mathcal{P}^{rs}_{\acute{b}n}-\frac{1}{15\sqrt{3}}\mathcal{P}^{\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{rs}_{\acute{b}k}+\frac{\sqrt{2}}{15}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}^{rs}_{\acute{b}k}\right]\mathsf{H}^{ijk}_{rs}\right\}\qquad(4.13)$$

5. Vector-Spinor Couplings in SO(10) Model Building

In addition to the cubic couplings in the superpotential involving the $144+\overline{144}$ multiplets given in the preceding section, their gauge couplings with vector mutiplets in the singlet representation and with an adjoint of SO(10) representation are given in the appendix B. Also given in the appendices are the quartic self-couplings of the $144+\overline{144}$ multiplets which are needed for spontaneous breaking of SO(10) and couplings of 10-plet of matter fields with $144+\overline{144}$ plet of Higgs which are needed to generate Yukawa couplings and quark-lepton mass textures. In this section we illustrate the use of vector-spinor couplings for further development of $SO(10)$ model building discussed in ref. [3]. In particular, we discuss the breaking of $SO(10)$ group down to the Standard Model group, doublet-triplet splitting, mass growth of quarks and leptons, and baryon and lepton violating dimension five operators. In ref. [3] it was shown that breaking of SO(10) to the Standard Model gauge group can be accomplished in one step using $160+\overline{160}$ multiplet. In the following we give a simpler illustration of how this comes about. This simpler example includes in the superpotential just the multiplets $144\times\overline{144}$ and only three terms, including the mass term and interaction terms mediated by 45 and 210 so that

$$\mathsf{W}=\mathsf{W}^{(\overline{144}_H\times 144_H)}+\mathsf{W}^{(\overline{144}_H\times 144_H)_{45}(\overline{144}_H\times 144_H)_{45}}+\mathsf{W}^{(\overline{144}_H\times 144_H)_{210}(\overline{144}_H\times 144_H)_{210}}\qquad(5.1)$$

Explicit forms of these couplings are

$$\mathsf{W}^{(\overline{144}_H\times 144_H)}=M<\Upsilon^*_{(-)\mu}|B|\Upsilon_{(+)\mu}>$$
$$\mathsf{W}^{(\overline{144}_H\times 144_H)_{45}(\overline{144}_H\times 144_H)_{45}}=\frac{\Lambda_{45}}{M'}<\Upsilon^*_{(-)\mu}|B\Sigma_{\rho\lambda}|\Upsilon_{(+)\mu}>\cdot<\Upsilon^*_{(-)\nu}|B\Sigma_{\rho\lambda}|\Upsilon_{(+)\nu}>$$
$$\mathsf{W}^{(\overline{144}_H\times 144_H)_{210}(\overline{144}_H\times 144_H)_{210}}=\frac{\Lambda_{210}}{M'}<\Upsilon^*_{(-)\mu}|B\Gamma_{[\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\xi]}|\Upsilon_{(+)\mu}>$$
$$\cdot<\Upsilon^*_{(-)\nu}|B\Gamma_{[\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\xi]}|\Upsilon_{(+)\nu}>\qquad(5.2)$$

Refer to the appendix C for the complete evaluation of the above quartic interactions in terms of $SU(5)$ fields.

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5.1 One step breaking of SO(10) GUT symmetry

The terms that contribute to one step breaking of GUT symmetry so that $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$ are

$$\mathsf{W}_{GS} = M \mathcal{Q}^i_j \mathcal{P}^j_i + \alpha_1 \mathcal{Q}^i_j \mathcal{P}^j_i \mathcal{Q}^k_l \mathcal{P}^l_k + \alpha_2 \mathcal{Q}^i_k \mathcal{P}^k_j \mathcal{Q}^j_l \mathcal{P}^l_i \tag{5.3}$$

where

$$\alpha_1 = \frac{1}{M'} \left(-\Lambda_{45} + \frac{1}{6} \Lambda_{210} \right), \quad \alpha_2 = \frac{1}{M'} \left(-4\Lambda_{45} - \Lambda_{210} \right) \tag{5.4}$$

For symmetry breaking we invoke the following vacuum expectation values (VEV's)

$$< \mathcal{Q}^i_j >= q \; diag(2,2,2,-3,-3), \quad < \mathcal{P}^i_j >= p \; diag(2,2,2,-3,-3) \tag{5.5}$$

and together with the minimization of W_{GS}, we find

$$\frac{MM'}{qp} = 116\lambda_{45} + 4\lambda_{210} \tag{5.6}$$

The D-flatness condition $< 144 >=< \overline{144} >$ gives $q = p$. With the above VEV, spontaneous breaking of $SO(10)$ occurs down to the Standard Model group. We note once again that in ref. [3] the gauge symmetry breaking was accomplished by use of $160 + \overline{160}$ while here are able to get a full breakdown of SO(10) to the Standard Model gauge group in one step just with $144 + \overline{144}$.

5.2 Doublet triplet splitting

As discussed in ref. [3]in the scenario with one step breaking of SO(10) both Higgs doublets and the Higgs triplets will be heavy. However, it is possible to get a pair of light Higgs doublets by fine tuning, a procedure which is justified in the context of landscape scenarios as discussed in ref. [3]. Here we illustrate this explicitly for the case of the superpotential of Eq.(5.1). To this end we collect the relevant terms using mixed $SO(10)$ and $SU(5)$ indices:

$$\begin{aligned} \mathsf{W}_{DT} = M \left(\mathbf{Q}_\mu \mathbf{P}_\mu - \frac{1}{2} \mathbf{Q}_{ij\mu} \mathbf{P}^{ij}_\mu \right) + \frac{\Lambda_{45}}{M'} \Big(& 8 \mathbf{Q}^i_\mu \mathbf{P}_{j\mu} \mathbf{Q}_{ik\nu} \mathbf{P}^{kj}_\nu + \mathbf{Q}^i_\mu \mathbf{P}_{i\mu} \mathbf{Q}_{jk\nu} \mathbf{P}^{jk}_\nu \\ & + 6 \mathbf{Q}_\mu \mathbf{P}_\mu \mathbf{Q}^i_\nu \mathbf{P}_{i\nu} \Big) + \frac{\Lambda_{210}}{M'} \Big(-\frac{2}{3} \mathbf{Q}^i_\mu \mathbf{P}_{j\mu} \mathbf{Q}_{ik\nu} \mathbf{P}^{kj}_\nu - \frac{1}{6} \mathbf{Q}^i_\mu \mathbf{P}_{i\mu} \mathbf{Q}_{jk\nu} \mathbf{P}^{jk}_\nu \\ & - \frac{1}{3} \mathbf{Q}_\mu \mathbf{P}_\mu \mathbf{Q}^i_\nu \mathbf{P}_{i\nu} - \frac{8}{3} \mathbf{Q}_\mu \mathbf{P}_{i\mu} \mathbf{Q}^i_\nu \mathbf{P}_\nu \Big) \end{aligned} \tag{5.7}$$

when expanded in purely $SU(5)$ indices, we get,

$$\begin{aligned} \mathsf{W}_{DT} = & \left[M + \frac{1}{M'} \left(6\Lambda_{45} - \frac{1}{3} \Lambda_{210} \right) < \mathbf{Q}^m_n >< \mathbf{P}^n_m > \right] \left[\mathbf{Q}_i \mathbf{P}^i + \mathbf{Q}^i \mathbf{P}_i \right] \\ & - \frac{8}{3} \frac{\Lambda_{210}}{M'} < \mathbf{Q}^i_m >< \mathbf{P}^m_j > \mathbf{Q}_i \mathbf{P}^j \\ & \left[-\frac{1}{2} M + \frac{1}{M'} \left(\Lambda_{45} - \frac{1}{6} \Lambda_{210} \right) < \mathbf{Q}^m_n >< \mathbf{P}^n_m > \right] \left[\mathbf{Q}^k_{ij} + \frac{1}{4} \left(\delta^k_i \mathbf{Q}_j - \delta^k_j \mathbf{Q}_i \right) \right] \end{aligned}$$

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$$
\times \left[\mathbf{P}_k^{ij} + \frac{1}{4}\left(\delta_k^i \mathbf{P}^j - \delta_k^j \mathbf{P}^i\right)\right]
$$

$$
+ \left[\frac{1}{M'}\left(\Lambda_{45} - \frac{1}{6}\Lambda_{210}\right) < \mathbf{Q}_m^i >< \mathbf{P}_j^m >\right] \left[\mathbf{Q}_{il}^k + \frac{1}{4}\left(\delta_i^k \mathbf{Q}_l - \delta_l^k \mathbf{Q}_i\right)\right]
$$

$$
\times \left[\mathbf{P}_k^{lj} + \frac{1}{4}\left(\delta_k^l \mathbf{P}^j - \delta_k^j \mathbf{P}^l\right)\right] \tag{5.8}
$$

Note that in addition to the pairs of doublets: $(\mathbf{Q}_\alpha, \mathbf{P}^\alpha)$, $(\mathbf{Q}^\alpha, \mathbf{P}_\alpha)$ $(\alpha, \beta, \gamma = 4, 5)$ and pairs of triplets: $(\mathbf{Q}_a, \mathbf{P}^a)$, $(\mathbf{Q}^a, \mathbf{P}_a)$ $(a, b, c = 1, 2, 3)$ there are also pairs of $SU(2)$ doublets and $SU(3)_C$ triplets and anti-triplets that reside in $\mathbf{Q}_{ij}^k$ and $\mathbf{P}_k^{ij}$. We denote them by $(\widetilde{\mathbf{Q}}_\alpha, \widetilde{\mathbf{P}}^\alpha)$, $(\widetilde{\mathbf{Q}}_a, \widetilde{\mathbf{P}}^a)$, $(\widetilde{\mathbf{Q}}^a, \widetilde{\mathbf{P}}_a)$. The mass matrix of the Higgs doublets is given by

$$
\begin{bmatrix}
\frac{3}{5}M + \frac{p^2}{M'}(\frac{666}{5}\Lambda_{45} - \frac{273}{10}\Lambda_{210}) & \frac{1}{4}\sqrt{\frac{15}{2}}\frac{p^2}{M'}(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}) & 0 \\
\frac{1}{4}\sqrt{\frac{15}{2}}\frac{p^2}{M'}(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}) & -\frac{1}{2}M + \frac{p^2}{M'}(-37\Lambda_{45} + \frac{7}{12}\Lambda_{210}) & 0 \\
0 & 0 & M + \frac{p^2}{M'}(180\Lambda_{45} - 10\Lambda_{210})
\end{bmatrix} \tag{5.9}
$$

where the columns are labelled by $(\mathcal{Q}_\alpha, \widetilde{\mathcal{Q}}_\alpha, \mathcal{P}_\alpha)$ and rows by $(\mathcal{P}^\alpha, \widetilde{\mathcal{P}}^\alpha, \mathcal{Q}^\alpha)$ where these are normalized fields as defined in appendix A. The triplet mass matrix in the basis where the columns are labelled by $(\mathcal{Q}_a, \widetilde{\mathcal{Q}}_a, \mathcal{P}_a, \widetilde{\mathcal{P}}_a)$ and rows by $(\mathcal{P}^a, \widetilde{\mathcal{P}}^a, \mathcal{Q}^a, \widetilde{\mathcal{Q}}^a)$ (where the fields are normalized fields as defined in appendix A) is given by

$$
\begin{bmatrix}
\frac{3}{5}M + \frac{p^2}{M'}(\frac{696}{5}\Lambda_{45} - \frac{257}{15}\Lambda_{210}) & \frac{1}{2}\sqrt{\frac{5}{2}}\frac{p^2}{M'}(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}) & 0 & 0 \\
\frac{1}{2}\sqrt{\frac{5}{2}}\frac{p^2}{M'}(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}) & -\frac{1}{2}M + \frac{p^2}{M'}(-12\Lambda_{45} - \frac{3}{2}\Lambda_{210}) & 0 & 0 \\
0 & 0 & M + \frac{p^2}{M'}(180\Lambda_{45} - 10\Lambda_{210}) & 0 \\
0 & 0 & 0 & -\frac{1}{2}M + \frac{p^2}{M'}(-42\Lambda_{45} + \Lambda_{210})
\end{bmatrix} \tag{5.10}
$$

It is clear from the above Higgs mass matrices that one needs to diagonalize in the Higgs doublet sub-sectors $(\widetilde{\mathcal{Q}}_\alpha, \widetilde{\mathcal{P}}^\alpha)$ and $(\mathcal{Q}_\alpha, \mathcal{P}^\alpha)$ and in the Higgs triplet subsectors $(\widetilde{\mathcal{Q}}_a, \widetilde{\mathcal{P}}^a)$ and $(\mathcal{Q}_a, \mathcal{P}^a)$. After, diagonalization we have the following pairs of doublets and triplets:

$$
\begin{aligned}
&\mathsf{D}_1: (\mathcal{Q}^\alpha, \mathcal{P}_\alpha), && \mathsf{T}_1: (\mathcal{Q}^a, \mathcal{P}_a) \\
&\mathsf{D}_2: (\mathcal{Q}'_\alpha, \mathcal{P}'^\alpha), && \mathsf{T}_2: (\mathcal{Q}'_a, \mathcal{P}'^a) \\
&\mathsf{D}_3: (\widetilde{\mathcal{Q}}'_\alpha, \widetilde{\mathcal{P}}'^\alpha), && \mathsf{T}_3: (\widetilde{\mathcal{Q}}'_a, \widetilde{\mathcal{P}}'^a) \\
& && \mathsf{T}_4: (\widetilde{\mathcal{Q}}^a, \widetilde{\mathcal{P}}_a)
\end{aligned} \tag{5.11}
$$

The prime fields above are expressed in terms of the original ones through the following transformation matrices

$$
\begin{bmatrix} (\mathcal{Q}'_a, \mathcal{P}'^a) \\ (\widetilde{\mathcal{Q}}'_a, \widetilde{\mathcal{P}}'^a) \end{bmatrix} = \begin{bmatrix} \cos\vartheta_{\mathsf{T}} & \sin\vartheta_{\mathsf{T}} \\ -\sin\vartheta_{\mathsf{T}} & \cos\vartheta_{\mathsf{T}} \end{bmatrix} \begin{bmatrix} (\mathcal{Q}_a, \mathcal{P}^a) \\ (\widetilde{\mathcal{Q}}_a, \widetilde{\mathcal{P}}^a) \end{bmatrix} \tag{5.12}
$$

$$
\begin{bmatrix} (\mathcal{Q}'_\alpha, \mathcal{P}'^\alpha) \\ (\widetilde{\mathcal{Q}}'_\alpha, \widetilde{\mathcal{P}}'^\alpha) \end{bmatrix} = \begin{bmatrix} \cos\vartheta_{\mathsf{D}} & \sin\vartheta_{\mathsf{D}} \\ -\sin\vartheta_{\mathsf{D}} & \cos\vartheta_{\mathsf{D}} \end{bmatrix} \begin{bmatrix} (\mathcal{Q}_\alpha, \mathcal{P}^\alpha) \\ (\widetilde{\mathcal{Q}}_\alpha, \widetilde{\mathcal{P}}^\alpha) \end{bmatrix} \tag{5.13}
$$

where

$$
\tan\vartheta_{\mathsf{T}} = \frac{1}{\mathsf{t}_3}\left(\mathsf{t}_2 + \sqrt{\mathsf{t}_2{}^2 + \mathsf{t}_3{}^2}\right), \quad \tan\vartheta_{\mathsf{D}} = \frac{1}{\mathsf{d}_3}\left(\mathsf{d}_2 + \sqrt{\mathsf{d}_2{}^2 + \mathsf{d}_3{}^2}\right) \tag{5.14}
$$

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and that

$$\begin{aligned}
\mathsf{d}_1 &= \frac{1}{10}M + \frac{p^2}{M'}\left(\frac{481}{5}\Lambda_{45} - \frac{1603}{60}\Lambda_{210}\right)\\
\mathsf{d}_2 &= -\frac{11}{10}M + \frac{p^2}{M'}\left(-\frac{851}{5}\Lambda_{45} + \frac{1673}{60}\Lambda_{210}\right)\\
\mathsf{d}_3 &= \frac{1}{2}\sqrt{\frac{15}{2}}\frac{p^2}{M'}\left(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}\right)\\
\mathsf{t}_1 &= \frac{1}{10}M + \frac{p^2}{M'}\left(\frac{636}{5}\Lambda_{45} - \frac{559}{30}\Lambda_{210}\right)\\
\mathsf{t}_2 &= -\frac{11}{10}M + \frac{p^2}{M'}\left(-\frac{756}{5}\Lambda_{45} + \frac{469}{30}\Lambda_{210}\right)\\
\mathsf{t}_3 &= \sqrt{\frac{5}{2}}\frac{p^2}{M'}\left(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}\right)
\end{aligned} \tag{5.15}$$

The mass eigenvalues are found to be

$$\begin{aligned}
M_{\mathsf{D}_2,\mathsf{D}_3} &= \frac{1}{2}\left(\mathsf{d}_1 \pm \sqrt{{\mathsf{d}_2}^2 + {\mathsf{d}_3}^2}\right)\\
M_{\mathsf{T}_2,\mathsf{T}_3} &= \frac{1}{2}\left(\mathsf{t}_1 \pm \sqrt{{\mathsf{t}_2}^2 + {\mathsf{t}_3}^2}\right)
\end{aligned} \tag{5.16}$$

and of course

$$\begin{aligned}
M_{\mathsf{D}_1} = M_{\mathsf{T}_1} &= M + \frac{p^2}{M'}(180\Lambda_{45} - 10\Lambda_{210})\\
M_{\mathsf{T}_4} &= -\frac{1}{2}M + \frac{p^2}{M'}(-42\Lambda_{45} + \Lambda_{210})
\end{aligned} \tag{5.17}$$

As an illustration we discuss in further detail the implication of the massless-ness condition for the doublet D_2. Here the condition $M_{\mathsf{D}_2} = 0$ together with the symmetry breaking condition, Eq. (5.6) gives a relationship among the parameters Λ_{45} and Λ_{210}

$$\left(\frac{539}{5}\Lambda_{45} - \frac{1579}{60}\Lambda_{210}\right)^2 = \left(-\frac{1489}{5}\Lambda_{45} + \frac{1409}{60}\Lambda_{210}\right)^2 + \frac{15}{8}\left(8\Lambda_{45} - \frac{2}{3}\Lambda_{210}\right)^2 \tag{5.18}$$

The two roots to the equations above are

$$\Lambda_{210} \approx 8.1\Lambda_{45}\,, \qquad \Lambda_{210} \approx -67.6\Lambda_{45} \tag{5.19}$$

Using the roots above the full doublet-triplet Higgs mass spectrum can now be computed. The results are summarized in the table below.

Massless Doublet D_2. M_D and M_T are in units of $\overline{M} \equiv \frac{p^2}{M'}\Lambda_{45}$

$\frac{M}{\overline{M}}$	$\frac{\Lambda_{210}}{\Lambda_{45}}$	M_{D_1}	M_{D_3}	M_{T_1}	M_{T_2}	M_{T_3}	M_{T_4}
148.4	8.1	247.4	-105.4	247.4	89.47	−98.4	−108.1
−154.4	−67.6	701.4	1887	701.6	1207	164.9	-32.4

We note that in the above only one pair of Higgs doublets is light while the remaining doublets and triplets are all heavy. Thus below the GUT scale one recovers the spectrum of MSSM. Further, one may carry out a similar analysis for the cases when $M_{D_1} = 0$ and $M_{D_3} = 0$.

5.3 Quark, lepton and neutrino masses

As pointed out in ref. [3] the quark, lepton and neutrino masses can arise from the quartic couplings involving two 16-plets of matter and two 144-plet of Higgs fields. Cubic Yukawa couplings arise when one of the two 144-plets is replaced by a VEV while mass terms arise when the remaining Higgs field in the cubic interaction develops a VEV. As an illustration of how this comes about in a concrete way we will consider the following quartic coupling for computing the masses of quarks and leptons: $(16\times16)_{120}(144\times144)_{120}$, $(16\times16)_{120}(\overline{144}\times\overline{144})_{120}$, $(16\times16)_{\overline{126}}(144\times144)_{126}$. However, this subsection is to be treated as an independent one. That is we do not make use of the results of the previous subsections here.

The relevant terms in Eqs. (139), (141) and (144) that gives mass growth to quarks and leptons are

$$\begin{aligned}
\mathsf{W}^{(120)}_{mass} = \xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big[&-\frac{4}{3}\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kn}_{\acute{b}}\frac{4}{3}\mathcal{P}^{x\mathbf{T}}_{\acute{c}n}\mathcal{P}^{lm}_{\acute{d}x} - \frac{4}{3\sqrt{5}}\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kn}_{\acute{b}}\mathcal{P}^{l\mathbf{T}}_{\acute{c}n}\mathcal{P}^{m}_{\acute{d}} \\
&-\frac{16}{3}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}j}\mathcal{P}^{\mathbf{T}}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}i} - \frac{16}{3}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathcal{P}^{ij\mathbf{T}}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}j} \\
&-\frac{8}{3\sqrt{5}}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathcal{P}^{j\mathbf{T}}_{\acute{c}}\mathcal{P}^{i}_{\acute{d}j} + \frac{32}{15}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}j}\mathcal{P}^{j\mathbf{T}}_{\acute{c}}\mathcal{P}^{i}_{\acute{d}}\Big] \\
+\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big[&\frac{4}{3}\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}}\mathcal{Q}^{m}_{\acute{d}n} + \frac{16}{3}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{jk}_{\acute{b}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}x}\mathcal{Q}^{x}_{\acute{d}jk} \\
&-\frac{16}{3}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}x}\mathcal{Q}^{x}_{\acute{d}kj} + \frac{16}{3\sqrt{5}}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{jk}_{\acute{b}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}j}\mathcal{Q}_{\acute{d}k} \\
&+\frac{8}{3\sqrt{5}}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}j}\mathcal{Q}_{\acute{d}k} - \frac{16}{3}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}}\mathcal{Q}^{i}_{\acute{d}j}\Big]
\end{aligned} \tag{5.20}$$

and

$$\begin{aligned}
\mathsf{W}^{(\overline{126},126)}_{mass} = \varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big[&\frac{4}{15}\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}}\mathcal{Q}^{m}_{\acute{d}n} \\
&-\frac{4}{15}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}k}\left(\mathcal{Q}^{l\mathbf{T}}_{\acute{c}ij}\mathcal{Q}^{k}_{\acute{d}l} - \frac{1}{\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{c}j}\mathcal{Q}^{k}_{\acute{d}i}\right) \\
&+\frac{1}{15}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}j}\left(2\mathcal{Q}^{l\mathbf{T}}_{\acute{c}ik}\mathcal{Q}^{k}_{\acute{d}l} + \frac{1}{\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{c}k}\mathcal{Q}^{k}_{\acute{d}i}\right) \\
&+\frac{32}{5}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}}\mathcal{Q}^{i}_{\acute{d}j} + \frac{16}{15\sqrt{5}}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}}\mathcal{Q}_{\acute{d}i}\Big]
\end{aligned} \tag{5.21}$$

For completeness we identify the Standard Model particles as follows:

$$\begin{aligned}
\mathbf{M}_{\acute{a}} = \boldsymbol{\nu}^{\mathcal{C}}_{L\acute{a}}; \quad \mathbf{M}_{\acute{a}\alpha} = \mathbf{D}^{\mathcal{C}}_{L\acute{a}\alpha}; \quad \mathbf{M}^{\alpha\beta}_{\acute{a}} = \epsilon^{\alpha\beta\gamma}\mathbf{U}^{\mathcal{C}}_{L\acute{a}\gamma}; \quad \mathbf{M}_{\acute{a}4} = \mathbf{E}^{-}_{L\acute{a}} \\
\mathbf{M}^{4\alpha}_{\acute{a}} = \mathbf{U}^{\alpha}_{L\acute{a}}; \quad \mathbf{M}_{\acute{a}5} = \boldsymbol{\nu}_{L\acute{a}}; \quad \mathbf{M}^{5\alpha}_{\acute{a}} = \mathbf{D}^{\alpha}_{L\acute{a}}; \quad \mathbf{M}^{45}_{\acute{a}} = \mathbf{E}^{+}_{L\acute{a}}
\end{aligned} \tag{5.22}$$

where $\alpha,\beta,\gamma = 1,2,3$ are color indices and the superscript $^{\mathcal{C}}$ denotes charge conjugation. We adopt the convention that all particles are left handed(L).

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We now single out the terms that are candidates for Majorana and Dirac neutrinos, Type II see-saw mechanism, down-type and up-type quarks and charged leptons.

Candidates for Majorana Neutrinos:

$$\mathbf{M}_{\acute{a}}\mathbf{M}_{\acute{b}}\left\{\left[\frac{16}{15\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{i}_{\acute{c}}\mathcal{Q}_{\acute{d}i}\right\} \tag{5.23}$$

Candidates for Dirac Neutrinos:

$$\begin{aligned}\mathbf{M}_{\acute{a}i}\mathbf{M}_{\acute{b}}\Bigg\{&\left[\frac{8}{3\sqrt{5}}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{P}^{j}_{\acute{c}}\mathcal{P}^{i}_{\acute{d}j}+\left[\frac{16}{3}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{P}^{ij}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}j}\\&+\left[\frac{32}{5}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}+\frac{16}{3}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{j}_{\acute{c}}\mathcal{Q}^{i}_{\acute{d}j}\Bigg\}\end{aligned} \tag{5.24}$$

Candidates for Type II See-Saw Mechanism:

$$\mathbf{M}_{\acute{a}i}\mathbf{M}_{\acute{b}j}\left[-\frac{32}{15}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{P}^{i}_{\acute{c}}\mathcal{P}^{j}_{\acute{d}} \tag{5.25}$$

Candidates for Down-type Quarks and Charged Leptons:

$$\begin{aligned}&\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}_{\acute{b}j}\Bigg\{\left[-\frac{16}{3}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{P}^{k}_{\acute{c}i}\mathcal{P}_{\acute{d}k}\\&\quad+\left[\frac{1}{15\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}+\frac{8}{3\sqrt{5}}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}_{\acute{c}k}\mathcal{Q}^{k}_{\acute{d}j}\\&\quad+\left[\frac{2}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}-\frac{16}{3}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{l}_{\acute{c}ik}\mathcal{Q}^{k}_{\acute{d}l}\Bigg\}\\&+\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}_{\acute{b}k}\Bigg\{\frac{1}{\sqrt{5}}\left[\frac{4}{15\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}+\frac{16}{3\sqrt{5}}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}_{\acute{c}j}\mathcal{Q}^{k}_{\acute{d}i}\\&\quad+\left[-\frac{4}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}+\frac{16}{3}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{l}_{\acute{c}ij}\mathcal{Q}^{k}_{\acute{d}l}\Bigg\}\end{aligned} \tag{5.26}$$

Candidates for Up-type Quarks:

$$\begin{aligned}&\epsilon_{ijklm}\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\left\{\left[\frac{4}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}+\frac{4}{3}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{n}_{\acute{c}}\mathcal{Q}^{m}_{\acute{d}n}\right\}\\&+\epsilon_{ijklm}\mathbf{M}^{in}_{\acute{a}}\mathbf{M}^{jk}_{\acute{b}}\left\{\left[\frac{4}{3}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{P}^{p}_{\acute{c}n}\mathcal{P}^{lm}_{\acute{d}p}+\left[\frac{4}{3\sqrt{5}}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{P}^{l}_{\acute{c}n}\mathcal{P}^{m}_{\acute{d}}\right\}\end{aligned} \tag{5.27}$$

Next we identify the $SU(3)_C \times U(1)_{em}$ conserving VEV's:

$$\begin{aligned}\begin{pmatrix}<\mathcal{Q}^{i}_{\acute{c}j}>\\<\mathcal{P}^{i}_{\acute{c}j}>\end{pmatrix}&=\begin{pmatrix}q_{\acute{c}}\\p_{\acute{c}}\end{pmatrix}diag(2,2,2,-3,-3)\\<\mathcal{Q}^{k}_{\acute{c}j5}>&=\frac{1}{2}\sqrt{\frac{3}{2}}<\widetilde{\mathcal{Q}}_{\acute{c}5}>\left(\frac{1}{4}\delta^{k}_{j}-\delta^{k}_{4}\delta^{4}_{j}\right)\\<\mathcal{P}^{j5}_{\acute{c}k}>&=\frac{1}{2}\sqrt{\frac{3}{2}}<\widetilde{\mathcal{P}}^{5}_{\acute{c}}>\left(\frac{1}{4}\delta^{j}_{k}-\delta^{j}_{4}\delta^{4}_{k}\right)\end{aligned} \tag{5.28}$$

To make further progress, we define the following mass parameters

$$\begin{array}{ll}
\alpha^{(120)}_{1\ \acute{a}\acute{b}} = 13\sqrt{\tfrac{2}{3}}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \widetilde{\mathcal{Q}}_{\acute{c}5} > q_{\acute{d}}, & \alpha^{(120)}_{2\ \acute{a}\acute{b}} = -\tfrac{8}{3\sqrt{5}}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{Q}_{\acute{c}5} > q_{\acute{d}} \\
\alpha^{(120)}_{3\ \acute{a}\acute{b}} = 16\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{P}_{\acute{c}5} > p_{\acute{d}}, & \alpha^{(126)}_{1\ \acute{a}\acute{b}} = -\tfrac{11}{5\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{Q}_{\acute{c}5} > q_{\acute{d}} \\
\alpha^{(126)}_{2\ \acute{a}\acute{b}} = -\tfrac{11}{20\sqrt{6}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \widetilde{\mathcal{Q}}_{\acute{c}5} > q_{\acute{d}} &
\end{array}$$

$$\begin{array}{ll}
a^{(120)}_{1\ \acute{a}\acute{b}} = -16\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{Q}^5_{\acute{c}} > q_{\acute{d}}, & a^{(120)}_{2\ \acute{a}\acute{b}} = -\tfrac{8}{\sqrt{5}}\sqrt{6}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{P}^5_{\acute{c}} > p_{\acute{d}} \\
a^{(120)}_{3\ \acute{a}\acute{b}} = 3\sqrt{3}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \widetilde{\mathcal{P}}^5_{\acute{c}} > p_{\acute{d}}, & a^{(126)}_{\acute{a}\acute{b}} = -\tfrac{32}{5}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{Q}^5_{\acute{c}} > q_{\acute{d}}
\end{array}$$

We now compute the down quark (M^{down}), charged lepton ($M^{electron}$), up quark (M^{up}), Dirac neutrino ($M^{Dirac\ \nu}$), RR type neutrino (M^{RR}) and LL type neutrino (M^{LL}) mass matrices in terms of the mass parameters defined above.

$$\begin{aligned} M^{down}_{\acute{a}\acute{b}} &= (A+B)_{\acute{a}\acute{b}} \\ M^{electron}_{\acute{a}\acute{b}} &= (A-3B)_{\acute{a}\acute{b}} \end{aligned} \tag{5.29}$$

where

$$A_{\acute{a}\acute{b}} = \left[\frac{59}{52}\alpha^{(120)}_1 + \frac{7}{2}\alpha^{(120)}_2 + \alpha^{(120)}_3 + \frac{16}{11}\alpha^{(126)}_1 + \frac{29}{22}\alpha^{(126)}_2\right]_{\acute{a}\acute{b}} \tag{5.30}$$

$$B_{\acute{a}\acute{b}} = \left[\frac{7}{52}\alpha^{(120)}_1 + \frac{5}{2}\alpha^{(120)}_2 + \frac{5}{11}\alpha^{(126)}_1 + \frac{7}{22}\alpha^{(126)}_2\right]_{\acute{a}\acute{b}} \tag{5.31}$$

and for the up quark and Dirac neutrino masses one has

$$M^{up}_{\acute{a}\acute{b}} = \left[a^{(120)}_1 + a^{(120)}_2 + a^{(120)}_3 + a^{(126)}\right]_{\acute{a}\acute{b}} \tag{5.32}$$

$$M^{Dirac\ \nu}_{\acute{a}\acute{b}} = \left[-a^{(120)}_1 - a^{(120)}_2 + \frac{5}{3}a^{(120)}_3 + \frac{1}{3}a^{(126)}\right]_{\acute{a}\acute{b}} \tag{5.33}$$

Majorana masses of RR and LL type for the neutrinos is given by

$$M^{RR}_{\acute{a}\acute{b}} = -\frac{16}{15\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{Q}^5_{\acute{c}} >< \mathcal{Q}_{\acute{d}5} > \tag{5.34}$$

$$M^{LL}_{\acute{a}\acute{b}} = \frac{32}{15}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \mathcal{P}^5_{\acute{c}} >< \mathcal{P}^5_{\acute{d}} > \tag{5.35}$$

For real model building one may now consider one at a time each of the doublets ($\mathcal{Q}_{\acute{a}\alpha}, \mathcal{P}^{\alpha}_{\acute{a}}$), ($\mathcal{Q}^{\alpha}_{\acute{a}}, \mathcal{P}_{\acute{a}\alpha}$), ($\widetilde{\mathcal{Q}}_{\acute{a}\alpha}, \widetilde{\mathcal{P}}^{\alpha}_{\acute{a}}$) massless and find the corresponding contribution to quark and lepton masses.

5.4 Baryon and lepton number violating dimension five operators

In supesymmetric theories with R parity the dominant proton decay arises from dimension five operators [13 – 15]. Here we look for baryon and lepton number violating dimension five operators in[1] $(16\times 16)_{10}\ (144\times 144)_{10}$, $(16\times 16)_{10}\left(\overline{144}\times\overline{144}\right)_{10}$ and $(16\times 16)_{\overline{126}}(144\times 144)_{126}$. We first collect all the terms from the three quartic couplings which contribute to baryon and lepton violating interactions. These are

$$\begin{aligned}
\mathsf{W} = \mathbf{M}^{ij}_{\acute{a}}\mathbf{M}_{\acute{b}j}\Bigg\{&\left[16\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right] < \mathcal{P}^{k}_{\acute{c}i} > \mathcal{P}_{\acute{d}k} + \left[\frac{1}{15\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} + \frac{8}{\sqrt{5}}\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}_{\acute{c}k} < \mathcal{Q}^{k}_{\acute{d}j} > \\
&+ \left[\frac{2}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} + 16\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{l}_{\acute{c}ik} < \mathcal{Q}^{k}_{\acute{d}l} > \Bigg\} \\
+\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}_{\acute{b}k}\Bigg\{&\left[\frac{4}{15\sqrt{5}}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}_{\acute{c}j} < \mathcal{Q}^{k}_{\acute{d}i} > + \left[-\frac{4}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{l}_{\acute{c}ij} < \mathcal{Q}^{k}_{\acute{d}l} > \Bigg\} \\
+\epsilon_{ijklm}\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\Bigg\{&\left[\frac{4}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} + 2\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right]\mathcal{Q}^{n}_{\acute{c}} < \mathcal{Q}^{m}_{\acute{d}n} > +2\left[\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right] < \mathcal{P}^{p}_{\acute{c}n} > \mathcal{P}^{nm}_{\acute{d}p} \\
&-\frac{1}{\sqrt{5}}\left[\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\right] < \mathcal{P}^{m}_{\acute{c}n} > \mathcal{P}^{n}_{\acute{d}}\Bigg\}
\end{aligned} \tag{5.36}$$

Expanding and collecting the relevant terms and inserting the triplet mass terms responsible for proton decay we find

$$\begin{aligned}
\mathsf{W}_{_{B\&L}} = {}& J_{(1)a}\mathcal{P}^{a} + K^{a}_{(1)}\mathcal{Q}_{a} + \mathrm{M}_{(\mathcal{Q}_a,\mathcal{P}^a)}\mathcal{Q}_{a}\mathcal{P}^{a} \\
&+J_{(2)a}\widetilde{\mathcal{P}}^{a} + K^{a}_{(2)}\widetilde{\mathcal{Q}}_{a} + \mathrm{M}_{(\widetilde{\mathcal{Q}}_a,\widetilde{\mathcal{P}}^a)}\widetilde{\mathcal{Q}}_{a}\widetilde{\mathcal{P}}^{a} \\
&+J^{a}_{(3)}\mathcal{P}_{a} + K_{(3)a}\mathcal{Q}^{a} + \mathrm{M}_{(\mathcal{Q}^a,\mathcal{P}_a)}\mathcal{Q}^{a}\mathcal{P}_{a} \\
&+K_{(4)a}\widetilde{\mathcal{Q}}^{a} + \mathrm{M}_{(\widetilde{\mathcal{Q}}^a,\widetilde{\mathcal{P}}_a)}\widetilde{\mathcal{Q}}^{a}\widetilde{\mathcal{P}}_{a}
\end{aligned} \tag{5.37}$$

where we have defined

$$\begin{aligned}
J_{(1)a} &= \left[-\frac{2}{\sqrt{5}}p\xi^{(10)(+)}_{\acute{a}\acute{b}}\right]\epsilon_{aijkl}\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}} \\
J_{(2)a} &= \left[10\sqrt{\frac{2}{3}}p\xi^{(10)(+)}_{\acute{a}\acute{b}}\right]\epsilon_{aijkl}\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}} \\
J^{a}_{(3)} &= \left[-32p\xi^{(10)(+)}_{\acute{a}\acute{b}}\right]\mathbf{M}_{\acute{a}i}\mathbf{M}^{ia}_{\acute{b}} \\
K^{a}_{(1)} &= \left[-\frac{16}{\sqrt{5}}q\zeta^{(10)(+)}_{\acute{a}\acute{b}} + \frac{2}{3\sqrt{5}}q\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right]\mathbf{M}_{\acute{a}\alpha}\mathbf{M}^{\alpha a}_{\acute{b}} \\
&\quad+\left[-\frac{16}{\sqrt{5}}q\zeta^{(10)(+)}_{\acute{a}\acute{b}} - \frac{2}{3\sqrt{5}}q\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right]\mathbf{M}_{\acute{a}b}\mathbf{M}^{ba}_{\acute{b}} \\
K^{a}_{(2)} &= \left[80\sqrt{\frac{2}{3}}q\zeta^{(10)(+)}_{\acute{a}\acute{b}} - \frac{2}{15}\sqrt{\frac{2}{3}}q\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right]\mathbf{M}_{\acute{a}\alpha}\mathbf{M}^{\alpha a}_{\acute{b}} \\
&\quad+\left[80\sqrt{\frac{2}{3}}q\zeta^{(10)(+)}_{\acute{a}\acute{b}} + \frac{2}{15}\sqrt{\frac{2}{3}}q\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right]\mathbf{M}_{\acute{a}b}\mathbf{M}^{ba}_{\acute{b}}
\end{aligned}$$

[1]For a complete analysis see ref. [4].

$$K_{(3)a} = \left[4q\zeta^{(10)(+)}_{\acute{a}\acute{b}} + \frac{8}{15}q\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right]\epsilon_{aijkl}\mathbf{M}^{ij}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}$$
$$K_{(4)a} = \left[-\frac{4\sqrt{2}}{15}q\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right]\mathbf{M}_{\acute{a}a}\mathbf{M}^{\alpha\beta}_{\acute{b}}\epsilon_{\alpha\beta} \tag{5.38}$$

Integrating out the Higgs triplet fields in Eq.(5.37) and expanding the results in Standard Model particle states, we get

$$\begin{aligned}
\mathsf{W}^{dim-5}_{B\&L} = 128pq &\left\{\left(\frac{1}{5\mathrm{M}_{(\mathcal{Q}_a,\mathcal{P}^a)}} + \frac{50}{3\mathrm{M}_{(\widetilde{\mathcal{Q}}_a,\widetilde{\mathcal{P}}^a)}}\right)\xi^{(10)(+)}_{\acute{a}\acute{b}}\zeta^{(10)(+)}_{\acute{c}\acute{d}}\right.\\
&\left.-\frac{4}{\mathrm{M}_{(\mathcal{Q}^a,\mathcal{P}_a)}}\left(\zeta^{(10)(+)}_{\acute{a}\acute{b}} + \frac{2}{15}\varrho^{(126,\overline{126})(+)}_{\acute{a}\acute{b}}\right)\xi^{(10)(+)}_{\acute{c}\acute{d}}\right\}\\
\times &\left[\epsilon_{abc}\mathbf{U}^a_{L\acute{a}}\mathbf{D}^b_{L\acute{b}}\left(\mathbf{E}^-_{L\acute{c}}\mathbf{U}^c_{L\acute{d}} + \boldsymbol{\nu}_{L\acute{c}}\mathbf{D}^c_{L\acute{d}}\right) + 2\epsilon^{abc}\mathbf{U}^{\mathcal{C}}_{L\acute{a}a}\mathbf{E}^+_{L\acute{b}}\mathbf{D}^{\mathcal{C}}_{L\acute{c}b}\mathbf{U}^{\mathcal{C}}_{L\acute{d}c}\right]\\
&-\frac{16}{3}pq\left\{\left(\frac{1}{5\mathrm{M}_{(\mathcal{Q}_a,\mathcal{P}^a)}} + \frac{2}{3\mathrm{M}_{(\widetilde{\mathcal{Q}}_a,\widetilde{\mathcal{P}}^a)}}\right)\xi^{(10)(+)}_{\acute{a}\acute{b}}\varrho^{(126,\overline{126})(+)}_{\acute{c}\acute{d}}\right\}\\
\times &\left[\epsilon_{abc}\mathbf{U}^a_{L\acute{a}}\mathbf{D}^b_{L\acute{b}}\left(\mathbf{E}^-_{L\acute{c}}\mathbf{U}^c_{L\acute{d}} + \boldsymbol{\nu}_{L\acute{c}}\mathbf{D}^c_{L\acute{d}}\right) - 2\epsilon^{abc}\mathbf{U}^{\mathcal{C}}_{L\acute{a}a}\mathbf{E}^+_{L\acute{b}}\mathbf{D}^{\mathcal{C}}_{L\acute{c}b}\mathbf{U}^{\mathcal{C}}_{L\acute{d}c}\right]
\end{aligned} \tag{5.39}$$

The above analysis shows that there are different varieties of contributions to the dimension five operator than one encounters in the minimal SO(10) model. Specifically, here one has contributions from the HIggs triplets and anti-triplets arising from the 45 and $\overline{45}$ SU(5) components of 144 and $\overline{144}$. The above raises the possibility of cancellations among various contributions allowing for the enhancement of the proton lifetime [4].

6. Conclusions

In this paper we have given an analysis of the couplings of the $144+\overline{144}$ multiplets. The 144 multiplet is interesting since it allows for the breaking of $SO(10)$ symmetry in a single step down to the Standard Model gauge group symmetry $SU(3)_C\times SU(2)_L\times U(1)_Y$. The 144 multiplet is a vector-spinor representation of $SO(10)$ with a constraint. The constraint is needed to reduce the components of the vector-spinor from 160 down to 144. These features make the analysis of the couplings of 144 and of $\overline{144}$ more complex than the couplings of ordinary spinors and tensor representations of $SO(10)$. In this paper we have utilized the techniques of the basic theorem to compute a variety of couplings involving the constrained vector-spinors: cubic couplings involving vector-spinors and tensors, self-couplings of the vector-spinors, and couplings of the vector-spinors with the 16 and $\overline{16}$ spinor representations of $SO(10)$. These couplings all enter in model building involving the spinors. Of course, the full set of couplings involving the vector-spinors are even larger, but these can also be computed using the techniques discussed here. We have shown how the breaking of SO(10) to the Standard Model gauge group can occur in a single step just one one pair of $144+\overline{144}$ plet of Higgs. We have also given illustrative examples of how Yukawa couplings, quark-lepton masses, and Dirac and Majorana neutrino masses arise from the couplings involving the 144 plet of Higgs. Finally we have exhibited how the baryon and lepton number violating interactions arise from the matter and 144 -plet couplings. The computations of the couplings given in section 4, and in appendices B–G and well as the analysis of section 5 constitute the new results of this paper. It is hoped that the techniques and the results presented here will be helpful in further development of model building involving the vector-spinor representation.

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A. Irreducible decompositions, normalization conditions, and notation

In this appendix we discuss the $SU(5)$ irreducible decomposition of the fields in $160+\overline{160}$ plets, normalize the $SU(5)$ fields contained in $144+\overline{144}$ plets, few remarks about the notation used in the paper and finally give the $SU(2)\times SU(3)$ decomposition of the 45 and $\overline{45}$ $SU(5)$ multiplets. The $SU(5)$ fields in $160+\overline{160}$ plets (see Eqs. (11) and (12)) are extracted from the reducible fields appearing in eqs. (3.1) and (3.2) as follows:

$$
\begin{aligned}
100=\overline{50}+50: &\quad \mathbf{P}_{\mu}^{ij}=\left(\mathbf{P}_{c_k}^{ij},\mathbf{P}_{\overline{c}_k}^{ij}\right)\equiv\left(\mathbf{R}^{[ij]k},\mathbf{R}_k^{[ij]}\right)\\
\overline{100}=\overline{50}+50: &\quad \mathbf{Q}_{\mu ij}=\left(\mathbf{Q}_{ijc_k},\mathbf{Q}_{ij\overline{c}_k}\right)\equiv\left(\mathbf{S}_{[ij]}^{k},\mathbf{S}_{[ij]k}\right)\\
\overline{50}=25+\overline{25}: &\quad \mathbf{P}_{i\mu}=\left(\mathbf{P}_{ic_k},\mathbf{P}_{i\overline{c}_k}\right)\equiv\left(\mathbf{R}_i^k,\mathbf{R}_{ik}\right)\\
50=25+25: &\quad \mathbf{Q}_{\mu}^{i}=\left(\mathbf{Q}_{\overline{c}_k}^{i},\mathbf{Q}_{c_k}^{i}\right)\equiv\left(\mathbf{S}_k^i,\mathbf{S}^{ik}\right)\\
10=\overline{5}+5: &\quad \mathbf{P}_{\mu}=\left(\mathbf{P}_{c_k},\mathbf{P}_{\overline{c}_k}\right)\equiv\left(\mathbf{P}^k,\mathbf{P}_k\right)\\
\overline{10}=\overline{5}+5: &\quad \mathbf{Q}_{\mu}=\left(\mathbf{Q}_{c_k},\mathbf{Q}_{\overline{c}_k}\right)\equiv\left(\mathbf{Q}^k,\mathbf{Q}_k\right)
\end{aligned}
$$

$$
\begin{aligned}
50=45+5: &\quad \mathbf{R}_k^{[ij]}=\mathbf{P}_k^{ij}+\frac{1}{4}\left(\delta_k^j\widehat{\mathbf{P}}^i-\delta_k^i\widehat{\mathbf{P}}^j\right)\\
\overline{50}=\overline{45}+\overline{5}: &\quad \mathbf{S}_{[jk]}^{i}=\mathbf{Q}_{jk}^{i}+\frac{1}{4}\left(\delta_k^i\widehat{\mathbf{Q}}_j-\delta_j^i\widehat{\mathbf{Q}}_k\right)\\
\overline{50}=\overline{40}+\overline{10}: &\quad \mathbf{R}^{[ij]k}=\epsilon^{ijlmn}\mathbf{P}_{lmn}^{k}+\epsilon^{ijklm}\widehat{\mathbf{P}}_{lm}\\
50=40+10: &\quad \mathbf{S}_{[ij]k}=\epsilon_{ijlmn}\mathbf{Q}_k^{lmn}+\epsilon_{ijklm}\widehat{\mathbf{Q}}^{lm}\\
25=24+1: &\quad \mathbf{R}_j^i=\mathbf{P}_j^i+\frac{1}{5}\delta_j^i\widehat{\mathbf{P}}\\
25=24+1: &\quad \mathbf{S}_j^i=\mathbf{Q}_j^i+\frac{1}{5}\delta_j^i\widehat{\mathbf{Q}}\\
\overline{25}=\overline{10}+\overline{15}: &\quad \mathbf{R}_{ij}=\frac{1}{2}\left(\mathbf{P}_{ij}+\mathbf{P}_{ij}^{(S)}\right)
\end{aligned}
$$

$$25 = 10 + 15: \quad \mathbf{S}^{ij} = \frac{1}{2}\left(\mathbf{Q}^{ij} + \mathbf{Q}^{ij}_{(S)}\right) \tag{A.1}$$

To normalize the $SU(5)$ fields contained in the tensor, $|\Upsilon_{(\pm)\mu}>$, we carry out a field redefinition

$$\begin{gathered}\{\overline{5}\}: \quad \mathbf{P}_i = \mathcal{P}_i, \qquad \{5\}: \quad \mathbf{P}^i = \frac{2}{\sqrt{5}}\mathcal{P}^i, \qquad \{\overline{10}\}: \quad \mathbf{P}_{ij} = \sqrt{\frac{6}{5}}\mathcal{P}_{ij} \\ \{\overline{15}\}: \quad \mathbf{P}^{(S)}_{ij} = \sqrt{2}\mathcal{P}^{(S)}_{ij}, \qquad \{24\}: \quad \mathbf{P}^i_j = \mathcal{P}^i_j, \qquad \{\overline{40}\}: \quad \mathbf{P}^l_{ijk} = \frac{1}{6}\mathcal{P}^l_{ijk} \\ \{45\}: \quad \mathbf{P}^{ij}_k = \mathcal{P}^{ij}_k\end{gathered} \tag{A.2}$$

$$\begin{gathered}\{5\}: \quad \mathbf{Q}^i = \mathcal{Q}^i, \qquad \{\overline{5}\}: \quad \mathbf{Q}_i = \frac{2}{\sqrt{5}}\mathcal{Q}_i, \qquad \{10\}: \quad \mathbf{Q}^{ij} = \sqrt{\frac{6}{5}}\mathcal{Q}^{ij} \\ \{15\}: \quad \mathbf{Q}^{ij}_{(S)} = \sqrt{2}\mathcal{Q}^{ij}_{(S)}, \qquad \{24\}: \quad \mathbf{Q}^i_j = \mathcal{Q}^i_j, \qquad \{40\}: \quad \mathbf{Q}^{ijk}_l = \frac{1}{6}\mathcal{Q}^{ijk}_l \\ \{\overline{45}\}: \quad \mathbf{Q}^k_{ij} = \mathcal{Q}^k_{ij}\end{gathered} \tag{A.3}$$

In terms of the normalized fields, the kinetic energy of the 144 and $\overline{144}$:
$-<\partial_A\Upsilon_{(\pm)\mu}|\partial^A\Upsilon_{(\pm)\mu}>$ takes the form

$$\begin{aligned}\mathsf{L}^{144}_{kin} &= -\partial_A\mathcal{P}^{i\dagger}\partial^A\mathcal{P}_i - \partial_A\mathcal{P}^\dagger_i\partial^A\mathcal{P}_i - \frac{1}{2!}\partial_A\mathcal{P}^{ij\dagger}\partial^A\mathcal{P}_{ij} \\ &-\frac{1}{2!}\partial_A\mathcal{P}^{ij\dagger}_{(S)}\partial^A\mathcal{P}^{(S)}_{ij} - \partial_A\mathcal{P}^{i\dagger}_j\partial^A\mathcal{P}^j_i - \frac{1}{3!}\partial_A\mathcal{P}^{ijk\dagger}_l\partial^A\mathcal{P}^l_{ijk} \\ &-\frac{1}{2!}\partial_A\mathcal{P}^{k\dagger}_{ij}\partial^A\mathcal{P}^{ij}_k\end{aligned} \tag{A.4}$$

$$\begin{aligned}\mathsf{L}^{\overline{144}}_{kin} &= -\partial_A\mathcal{Q}^\dagger_i\partial^A\mathcal{Q}^i - \partial_A\mathcal{Q}^{i\dagger}\partial^A\mathcal{Q}_i - \frac{1}{2!}\partial_A\mathcal{Q}^\dagger_{ij}\partial^A\mathcal{Q}^{ij} \\ &-\frac{1}{2!}\partial_A\mathcal{Q}^{(S)\dagger}_{ij}\partial^A\mathcal{Q}^{ij}_{(S)} - \partial_A\mathcal{Q}^{i\dagger}_j\partial^A\mathcal{Q}^j_i - \frac{1}{3!}\partial_A\mathcal{Q}^{l\dagger}_{ijk}\partial^A\mathcal{Q}^{ijk}_l \\ &-\frac{1}{2!}\partial_A\mathcal{Q}^{ij\dagger}_k\partial^A\mathcal{Q}^k_{ij}\end{aligned} \tag{A.5}$$

where $A = 0, 1, 2, 3$ represents the Lorentz index.

For ease of reference we give below the notations that will be used in much of the paper(i) The set of indices $(\mathcal{U},\mathcal{U}')...(\mathcal{Z},\mathcal{Z}')$ run over several Higgs representations of the same kind, (ii) $\mathcal{M}^{(\cdot)}$ represents mass matrices $h^{(\cdot)}$, $\bar{h}^{(\cdot)}$, $f^{(\cdot)}$, $\bar{f}^{(\cdot)}$, $g^{(\cdot)}$, $\bar{g}^{(\cdot)}$; $k^{(\cdot)}$, $\bar{k}^{(\cdot)}$, $l^{(\cdot)}$, $\bar{l}^{(\cdot)}$ are constants, (iii) An antisymmetric product of four Γ's for example, is represented by $\Gamma_{[\mu}\Gamma_\nu\Gamma_\rho\Gamma_{\lambda]} = \frac{1}{4!}\sum_P(-1)^{\delta_P}\Gamma_{\mu_{P(1)}}\Gamma_{\nu_{P(2)}}\Gamma_{\rho_{P(3)}}\Gamma_{\lambda_{P(4)}}$ with $\sum_P$ denoting the sum over all permutations and δ_P takes on the value 0 and 1 for even and odd permutations respectively.

We discuss now the $SU(2)\times SU(3)$ decomposition of the 45 and $\overline{45}$ SU(5) multiplets used in model building in section 4. We denote the doublets that arise from these ($\widetilde{\mathbf{Q}}_\alpha$, $\widetilde{\mathbf{P}}^\alpha$), and the triplets by ($\widetilde{\mathbf{Q}}_a$, $\widetilde{\mathbf{P}}^a$), ($\widetilde{\mathbf{Q}}^a$, $\widetilde{\mathbf{P}}_a$). We exhibit the decompositions below.

$$\mathbf{Q}^b_{b\alpha} = -\mathbf{Q}^\beta_{\beta\alpha} = \widetilde{\mathbf{Q}}_\alpha, \quad \mathbf{P}^{b\alpha}_b = -\mathbf{P}^{\beta\alpha}_\beta = \widetilde{\mathbf{P}}^\alpha$$

$$
\begin{aligned}
\mathbf{Q}_{ba}^{b} = -\mathbf{Q}_{\beta a}^{\beta} = \widetilde{\mathbf{Q}}_{a}, \quad \mathbf{P}_{b}^{ba} = -\mathbf{P}_{\beta}^{\beta a} = \widetilde{\mathbf{P}}^{a} \\
\mathbf{Q}_{b\alpha}^{a} = \widetilde{\mathbf{Q}}_{b\alpha}^{a} + \frac{1}{3}\delta_{b}^{a}\widetilde{\mathbf{Q}}_{\alpha}, \quad \mathbf{P}_{b}^{a\alpha} = \widetilde{\mathbf{P}}_{b}^{a\alpha} + \frac{1}{3}\delta_{b}^{a}\widetilde{\mathbf{P}}^{\alpha}, \quad \widetilde{\mathbf{Q}}_{b\alpha}^{b} = 0 = \widetilde{\mathbf{P}}_{b}^{b\alpha} \\
\mathbf{Q}_{\beta a}^{\alpha} = \widetilde{\mathbf{Q}}_{\beta a}^{\alpha} - \frac{1}{2}\delta_{\beta}^{\alpha}\widetilde{\mathbf{Q}}_{a}, \quad \mathbf{P}_{\alpha}^{ab} = \widetilde{\mathbf{P}}_{\beta}^{\alpha a} - \frac{1}{2}\delta_{\beta}^{\alpha}\widetilde{\mathbf{P}}^{a}, \quad \widetilde{\mathbf{Q}}_{\alpha b}^{\alpha} = 0 = \widetilde{\mathbf{P}}_{\alpha}^{\alpha b} \\
\mathbf{Q}_{bc}^{a} = \widetilde{\mathbf{Q}}_{bc}^{a} + \frac{1}{2}\left(\delta_{b}^{a}\widetilde{\mathbf{Q}}_{c} - \delta_{c}^{a}\widetilde{\mathbf{Q}}_{b}\right), \quad \mathbf{P}_{c}^{ab} = \widetilde{\mathbf{P}}_{c}^{ab} + \frac{1}{2}\left(\delta_{c}^{a}\widetilde{\mathbf{P}}^{b} - \delta_{c}^{b}\widetilde{\mathbf{P}}_{a}\right), \quad \widetilde{\mathbf{Q}}_{ab}^{a} = 0 = \widetilde{\mathbf{P}}_{a}^{ab} \\
\mathbf{Q}_{\beta\gamma}^{\alpha} = \widetilde{\mathbf{Q}}_{\beta\gamma}^{\alpha} + \left(\delta_{\gamma}^{\alpha}\widetilde{\mathbf{Q}}_{\beta} - \delta_{\beta}^{\alpha}\widetilde{\mathbf{Q}}_{\gamma}\right), \quad \mathbf{P}_{\gamma}^{\alpha\beta} = \widetilde{\mathbf{P}}_{\gamma}^{\alpha\beta} + \left(\delta_{\gamma}^{\beta}\widetilde{\mathbf{P}}^{\alpha} - \delta_{\gamma}^{\alpha}\widetilde{\mathbf{P}}_{\beta}\right), \quad \widetilde{\mathbf{Q}}_{\alpha\beta}^{\alpha} = 0 = \widetilde{\mathbf{P}}_{\alpha}^{\alpha\beta} \\
\mathbf{Q}_{\alpha\beta}^{a} = \epsilon_{\alpha\beta}\widetilde{\mathbf{Q}}^{a}, \quad \mathbf{P}_{a}^{\alpha\beta} = \epsilon^{\alpha\beta}\widetilde{\mathbf{P}}_{a} \qquad \text{(A.6)}
\end{aligned}
$$

The kinetic energy of the 45 and $\overline{45}$ fields is given by

$$
\begin{aligned}
-\partial_A\mathbf{Q}_{ij}^{k}\partial^A\mathbf{Q}_{ij}^{k\dagger} - \partial_A\mathbf{P}_{k}^{ij}\partial^A\mathbf{P}_{k}^{ij\dagger} = -\partial_A\widetilde{\mathcal{Q}}_{\alpha}\partial^A\widetilde{\mathcal{Q}}_{\alpha}^{\dagger} - \partial_A\widetilde{\mathcal{Q}}_{a}\partial^A\widetilde{\mathcal{Q}}_{a}^{\dagger} - \partial_A\widetilde{\mathcal{Q}}^{a}\partial^A\widetilde{\mathcal{Q}}^{a\dagger} \\
-\partial_A\widetilde{\mathcal{P}}^{\alpha}\partial^A\widetilde{\mathcal{P}}^{\alpha\dagger} - \partial_A\widetilde{\mathcal{P}}^{a}\partial^A\widetilde{\mathcal{P}}^{a\dagger} - \partial_A\widetilde{\mathcal{P}}_{a}\partial^A\widetilde{\mathcal{P}}_{a}^{\dagger} - .... \qquad \text{(A.7)}
\end{aligned}
$$

so that the doublet and triplet fields are normalized according to

$$
\begin{aligned}
\widetilde{\mathbf{Q}}_{\alpha} = \frac{1}{2}\sqrt{\frac{3}{2}}\widetilde{\mathcal{Q}}_{\alpha}, \quad \widetilde{\mathbf{Q}}_{a} = \sqrt{\frac{1}{2}}\widetilde{\mathcal{Q}}_{a}, \quad \widetilde{\mathbf{Q}}^{a} = \frac{1}{\sqrt{2}}\widetilde{\mathcal{Q}}^{a} \\
\widetilde{\mathbf{P}}^{\alpha} = \frac{1}{2}\sqrt{\frac{3}{2}}\widetilde{\mathcal{P}}^{\alpha}, \quad \widetilde{\mathbf{P}}^{a} = \sqrt{\frac{1}{2}}\widetilde{\mathcal{P}}^{a}, \quad \widetilde{\mathbf{P}}_{a} = \frac{1}{\sqrt{2}}\widetilde{\mathcal{P}}_{a} \qquad \text{(A.8)}
\end{aligned}
$$

B. The gauge couplings of vector-spinors

In this appendix we compute the interactions of the 144 and $\overline{144}$ with gauge tensors 1 and 45.

B.1 The $\overline{\mathbf{144}}^{\dagger} \times \overline{\mathbf{144}} \times \mathbf{1}$ couplings

These couplings are given by

$$
\mathsf{L}_{++}^{(1)} = g_{\acute{a}\acute{b}}^{(1)} < \Upsilon_{(+)\acute{a}\mu}|\gamma^0\gamma^A|\Upsilon_{(+)\acute{b}\mu} > \Phi_{A\rho\sigma} \qquad \text{(B.1)}
$$

where $\gamma^A(A, B = 0-3)$ spans the Clifford algebra associated with the Lorentz group. An explicit analysis in the $SU(5)\times U(1)$ basis gives

$$
\begin{aligned}
\mathsf{L}_{++}^{(1)} = g_{\acute{a}\acute{b}}^{(1)}\Big\{\Big[\overline{\mathcal{P}}_{\acute{a}j}^{i}\gamma^A\mathcal{P}_{\acute{b}i}^{j} + \overline{\mathcal{P}}_{\acute{a}}^{ij}\gamma^A\mathcal{P}_{\acute{b}ij} + \frac{1}{2}\overline{\mathcal{P}}_{(S)\acute{a}}^{ij}\gamma^A\mathcal{P}_{\acute{b}ij}^{(S)} + \frac{1}{2}\overline{\mathcal{P}}_{\acute{a}ij}^{k}\gamma^A\mathcal{P}_{\acute{b}k}^{ij} \\
+\overline{\mathcal{P}}_{\acute{a}i}\gamma^A\mathcal{P}_{\acute{b}}^{i} + \frac{1}{6}\overline{\mathcal{P}}_{\acute{a}}^{i}\gamma^A\mathcal{P}_{\acute{b}i} + \frac{1}{2}\overline{\mathcal{P}}_{\acute{a}l}^{ijk}\gamma^A\mathcal{P}_{\acute{b}ijk}^{l}\Big]\mathsf{G}_A \qquad \text{(B.2)}
\end{aligned}
$$

B.2 The $\mathbf{144}^{\dagger} \times \mathbf{144} \times \mathbf{1}$ couplings

These couplings are given by

$$
\mathsf{L}_{--}^{(1)} = \bar{g}_{\acute{a}\acute{b}}^{(1)} < \Upsilon_{(-)\acute{a}\mu}|\gamma^0\gamma^A|\Upsilon_{(-)\acute{b}\mu} > \Phi_{A\rho\sigma} \qquad \text{(B.3)}
$$

An explicit analysis in the $SU(5)\times U(1)$ basis gives

$$
\begin{aligned}
\mathsf{L}^{(1)}_{--} = \bar{g}^{(1)}_{\acute{a}\acute{b}} \Bigg\{ \Bigg[& \overline{\mathcal{Q}}^{j}_{\acute{a}i}\gamma^A \mathcal{Q}^{i}_{\acute{b}j} + \overline{\mathcal{Q}}_{\acute{a}ij}\gamma^A \mathcal{Q}^{ij}_{\acute{b}} + \frac{1}{2}\overline{\mathcal{Q}}^{(S)}_{\acute{a}ij}\gamma^A \mathcal{Q}^{ij}_{(S)\acute{b}} + \frac{1}{2}\overline{\mathcal{Q}}^{ij}_{\acute{a}k}\gamma^A \mathcal{Q}^{k}_{\acute{b}ij} \\
& + \overline{\mathcal{Q}}^{i}_{\acute{a}}\gamma^A \mathcal{Q}_{\acute{b}i} + \frac{1}{6}\overline{\mathcal{Q}}_{\acute{a}i}\gamma^A \mathcal{Q}^{i}_{\acute{b}} + \frac{1}{2}\overline{\mathcal{Q}}^{l}_{\acute{a}ijk}\gamma^A \mathcal{Q}^{ijk}_{\acute{b}l} \Bigg] \mathsf{G}_A
\end{aligned}
\tag{B.4}
$$

B.3 The $\overline{\mathbf{144}}^{\dagger}\times\overline{\mathbf{144}}\times\mathbf{45}$ couplings

These couplings are defined by

$$
\mathsf{L}^{(45)}_{++} = \frac{1}{i}\frac{1}{2!} g^{(45)}_{\acute{a}\acute{b}} < \Upsilon_{(+)\acute{a}\mu}|\gamma^0\gamma^A\Sigma_{\rho\sigma}|\Upsilon_{(+)\acute{b}\mu} > \Phi_{A\rho\sigma}
\tag{B.5}
$$

An expansion in the $SU(5)\times U(1)$ basis using the Basic Theorem gives

$$
\begin{aligned}
\mathsf{L}^{(45)}_{++} = g^{(45)}_{\acute{a}\acute{b}} \Bigg\{ \Bigg[& -\frac{3}{\sqrt{5}}\overline{\mathcal{P}}^{i}_{\acute{a}j}\gamma^A\mathcal{P}^{j}_{\acute{b}i} - \frac{7}{10\sqrt{5}}\overline{\mathcal{P}}^{ij}_{\acute{a}}\gamma^A\mathcal{P}_{\acute{b}ij} - \frac{3}{2\sqrt{5}}\overline{\mathcal{P}}^{ij}_{(S)\acute{a}}\gamma^A\mathcal{P}^{(S)}_{\acute{b}ij} + \frac{1}{2\sqrt{5}}\overline{\mathcal{P}}^{k}_{\acute{a}ij}\gamma^A\mathcal{P}^{ij}_{\acute{b}k} \\
& + \frac{21}{5\sqrt{5}}\overline{\mathcal{P}}_{\acute{a}i}\gamma^A\mathcal{P}^{i}_{\acute{b}} - \sqrt{5}\overline{\mathcal{P}}^{i}_{\acute{a}}\gamma^A\mathcal{P}_{\acute{b}i} + \frac{1}{6\sqrt{5}}\overline{\mathcal{P}}^{ijk}_{\acute{a}l}\gamma^A\mathcal{P}^{l}_{\acute{b}ijk} \Bigg] \mathsf{G}_A \\
& + \Bigg[\frac{1}{\sqrt{2}}\overline{\mathcal{P}}^{k}_{\acute{a}}\gamma^A\mathcal{P}^{lm}_{\acute{b}k} + \frac{1}{\sqrt{10}}\overline{\mathcal{P}}^{l}_{\acute{a}}\gamma^A\mathcal{P}^{m}_{\acute{b}} + \frac{2}{\sqrt{15}}\overline{\mathcal{P}}^{lk}_{\acute{a}}\gamma^A\mathcal{P}^{m}_{\acute{b}k} + \frac{1}{\sqrt{2}}\overline{\mathcal{P}}^{klm}_{\acute{a}n}\gamma^A\mathcal{P}^{n}_{\acute{b}k} \\
& - \frac{1}{20\sqrt{3}}\epsilon^{ijklm}\overline{\mathcal{P}}_{\acute{a}i}\gamma^A\mathcal{P}_{\acute{b}jk} + \frac{1}{3\sqrt{10}}\epsilon^{ijklm}\overline{\mathcal{P}}_{\acute{a}n}\gamma^A\mathcal{P}^{n}_{\acute{b}ijk} - \frac{1}{4}\sqrt{\frac{3}{5}}\epsilon^{ijklm}\overline{\mathcal{P}}^{n}_{\acute{a}ij}\gamma^A\mathcal{P}_{\acute{b}nk} \\
& + \frac{1}{4}\epsilon^{ijklm}\overline{\mathcal{P}}^{n}_{\acute{a}ij}\gamma^A\mathcal{P}^{(S)}_{\acute{b}nk} \Bigg] \mathsf{G}_{Alm} \\
& + \Bigg[\frac{1}{\sqrt{2}}\overline{\mathcal{P}}^{k}_{\acute{a}lm}\gamma^A\mathcal{P}_{\acute{b}k} - \frac{1}{\sqrt{10}}\overline{\mathcal{P}}_{\acute{a}l}\gamma^A\mathcal{P}_{\acute{b}m} + \frac{2}{\sqrt{15}}\overline{\mathcal{P}}^{k}_{\acute{a}l}\gamma^A\mathcal{P}_{\acute{b}km} + \frac{1}{\sqrt{2}}\overline{\mathcal{P}}^{k}_{\acute{a}n}\gamma^A\mathcal{P}^{n}_{\acute{b}klm} \\
& - \frac{1}{20\sqrt{3}}\epsilon_{ijklm}\overline{\mathcal{P}}^{ij}_{\acute{a}}\gamma^A\mathcal{P}^{k}_{\acute{b}} + \frac{1}{3\sqrt{10}}\epsilon_{ijklm}\overline{\mathcal{P}}^{ijk}_{\acute{a}n}\gamma^A\mathcal{P}^{n}_{\acute{b}} + \frac{1}{4}\sqrt{\frac{3}{5}}\epsilon_{ijklm}\overline{\mathcal{P}}^{in}_{\acute{a}}\gamma^A\mathcal{P}^{jk}_{\acute{b}n} \\
& + \frac{1}{4}\epsilon_{ijklm}\overline{\mathcal{P}}^{in}_{(S)\acute{a}}\gamma^A\mathcal{P}^{jk}_{\acute{b}n} \Bigg] \mathsf{G}^{lm}_{A} \\
& + \Bigg[\sqrt{2}\overline{\mathcal{P}}^{l}_{\acute{a}ik}\gamma^A\mathcal{P}^{kj}_{\acute{b}l} - \frac{1}{\sqrt{10}}\overline{\mathcal{P}}^{j}_{\acute{a}ik}\gamma^A\mathcal{P}^{k}_{\acute{b}} + \frac{1}{\sqrt{10}}\overline{\mathcal{P}}_{\acute{a}k}\gamma^A\mathcal{P}^{kj}_{\acute{b}i} - \frac{3}{10\sqrt{2}}\overline{\mathcal{P}}_{ai}\gamma^A\mathcal{P}^{j}_{\acute{b}} \\
& + \frac{1}{\sqrt{2}}\overline{\mathcal{P}}^{jkl}_{\acute{a}m}\gamma^A\mathcal{P}^{m}_{\acute{b}kli} - \frac{1}{\sqrt{15}}\overline{\mathcal{P}}^{jkl}_{\acute{a}i}\gamma^A\mathcal{P}_{\acute{b}kl} - \frac{1}{\sqrt{15}}\overline{\mathcal{P}}^{kl}_{\acute{a}}\gamma^A\mathcal{P}^{j}_{\acute{b}ikl} - \frac{17}{15\sqrt{2}}\overline{\mathcal{P}}^{jk}_{\acute{a}}\gamma^A\mathcal{P}_{\acute{b}ki} \\
& + \sqrt{\frac{3}{10}}\overline{\mathcal{P}}^{jk}_{\acute{a}}\gamma^A\mathcal{P}^{(S)}_{\acute{b}ki} - \sqrt{\frac{3}{10}}\overline{\mathcal{P}}^{jk}_{(S)\acute{a}}\gamma^A\mathcal{P}_{\acute{b}ki} + \frac{1}{\sqrt{2}}\overline{\mathcal{P}}^{jk}_{(S)\acute{a}}\gamma^A\mathcal{P}^{(S)}_{\acute{b}ki} + \sqrt{2}\overline{\mathcal{P}}^{j}_{\acute{a}k}\gamma^A\mathcal{P}^{k}_{\acute{b}i} \Bigg] \mathsf{G}^{i}_{Aj} \Bigg\}
\end{aligned}
\tag{B.6}
$$

The barred matter fields are defined so that $\overline{\mathcal{P}}^{jk}_{\acute{a}} = \mathcal{P}^{jk\dagger}_{\acute{a}}\gamma^0$, etc..

B.4 The $\mathbf{144}^{\dagger}\times\mathbf{144}\times\mathbf{45}$ couplings

These gauge couplings are defined by

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$$\mathsf{L}^{(45)}_{--} = \frac{1}{i}\frac{1}{2!}\bar{g}^{(45)}_{\acute{a}\acute{b}} < \Upsilon_{(-)\acute{a}\mu}|\gamma^0\gamma^A\Sigma_{\rho\sigma}|\Upsilon_{(-)\acute{b}\mu} > \Phi_{A\rho\sigma} \tag{B.7}$$

An analysis in the $SU(5)\times U(1)$ basis using the Basic Theorem gives

$$\begin{aligned}
\mathsf{L}^{(45)}_{--} = \bar{g}^{(45)}_{\acute{a}\acute{b}} \Bigg\{ \Bigg[& \frac{3}{\sqrt{5}}\overline{\mathcal{Q}}^{j}_{\acute{a}i}\gamma^A\mathcal{Q}^{i}_{\acute{b}j} + \frac{7}{10\sqrt{5}}\overline{\mathcal{Q}}_{\acute{a}ij}\gamma^A\mathcal{Q}^{ij}_{\acute{b}} + \frac{3}{2\sqrt{5}}\overline{\mathcal{Q}}^{(S)}_{\acute{a}ij}\gamma^A\mathcal{Q}^{ij}_{(S)\acute{b}} - \frac{1}{2\sqrt{5}}\overline{\mathcal{Q}}^{ij}_{\acute{a}k}\gamma^A\mathcal{Q}^{k}_{\acute{b}ij} \\
& -\frac{21}{5\sqrt{5}}\overline{\mathcal{Q}}^{i}_{\acute{a}}\gamma^A\mathcal{Q}_{\acute{b}i} - \sqrt{5}\overline{\mathcal{Q}}_{\acute{a}i}\gamma^A\mathcal{Q}^{i}_{\acute{b}} - \frac{1}{6\sqrt{5}}\overline{\mathcal{Q}}^{l}_{\acute{a}ijk}\gamma^A\mathcal{Q}^{ijk}_{\acute{b}l}\Bigg]\mathsf{G}_A \\
+ \Bigg[& \frac{1}{\sqrt{2}}\overline{\mathcal{Q}}^{lm}_{\acute{a}k}\gamma^A\mathcal{Q}^{k}_{\acute{b}} + \frac{1}{\sqrt{10}}\overline{\mathcal{Q}}^{m}_{\acute{a}}\gamma^A\mathcal{Q}^{l}_{\acute{b}} + \frac{2}{\sqrt{15}}\overline{\mathcal{Q}}^{m}_{\acute{a}k}\gamma^A\mathcal{Q}^{lk}_{\acute{b}} + \frac{1}{\sqrt{2}}\overline{\mathcal{Q}}^{n}_{\acute{a}k}\gamma^A\mathcal{Q}^{klm}_{\acute{b}n} \\
& -\frac{1}{20\sqrt{3}}\epsilon^{ijklm}\overline{\mathcal{Q}}_{\acute{a}jk}\gamma^A\mathcal{Q}_{\acute{b}i} + \frac{1}{3\sqrt{10}}\epsilon^{ijklm}\overline{\mathcal{Q}}^{n}_{\acute{a}ijk}\gamma^A\mathcal{Q}_{\acute{b}n} - \frac{1}{4}\sqrt{\frac{3}{5}}\epsilon^{ijklm}\overline{\mathcal{Q}}_{\acute{a}nk}\gamma^A\mathcal{Q}^{n}_{\acute{b}ij} \\
& +\frac{1}{4}\epsilon^{ijklm}\overline{\mathcal{Q}}^{(S)}_{\acute{a}nk}\gamma^A\mathcal{Q}^{n}_{\acute{b}ij}\Bigg]\mathsf{G}_{Alm} \\
+ \Bigg[& \frac{1}{\sqrt{2}}\overline{\mathcal{Q}}_{\acute{a}k}\gamma^A\mathcal{Q}^{k}_{\acute{b}lm} - \frac{1}{\sqrt{10}}\overline{\mathcal{Q}}_{\acute{a}m}\gamma^A\mathcal{Q}_{\acute{b}l} + \frac{2}{\sqrt{15}}\overline{\mathcal{Q}}_{\acute{a}km}\gamma^A\mathcal{Q}^{k}_{\acute{b}l} + \frac{1}{\sqrt{2}}\overline{\mathcal{Q}}^{n}_{\acute{a}klm}\gamma^A\mathcal{Q}^{k}_{\acute{b}n} \\
& -\frac{1}{20\sqrt{3}}\epsilon_{ijklm}\overline{\mathcal{Q}}^{k}_{\acute{a}}\gamma^A\mathcal{Q}^{ij}_{\acute{b}} + \frac{1}{3\sqrt{10}}\epsilon_{ijklm}\overline{\mathcal{Q}}^{n}_{\acute{a}}\gamma^A\mathcal{Q}^{ijk}_{\acute{b}n} + \frac{1}{4}\sqrt{\frac{3}{5}}\epsilon_{ijklm}\overline{\mathcal{Q}}^{jk}_{\acute{a}n}\gamma^A\mathcal{Q}^{in}_{\acute{b}} \\
& +\frac{1}{4}\epsilon_{ijklm}\overline{\mathcal{Q}}^{jk}_{\acute{a}n}\gamma^A\mathcal{Q}^{in}_{(S)\acute{b}}\Bigg]\mathsf{G}^{lm}_{A} \\
+ \Bigg[& -\sqrt{2}\overline{\mathcal{Q}}^{kj}_{\acute{a}l}\gamma^A\mathcal{Q}^{l}_{\acute{b}ik} + \frac{1}{\sqrt{10}}\overline{\mathcal{Q}}^{k}_{\acute{a}}\gamma^A\mathcal{Q}^{j}_{\acute{b}ik} - \frac{1}{\sqrt{10}}\overline{\mathcal{Q}}^{kj}_{\acute{a}i}\gamma^A\mathcal{Q}_{\acute{b}k} + \frac{3}{10\sqrt{2}}\overline{\mathcal{Q}}^{j}_{\acute{a}}\gamma^A\mathcal{Q}_{\acute{b}i} \\
& -\frac{1}{\sqrt{2}}\overline{\mathcal{Q}}^{m}_{\acute{a}ikl}\gamma^A\mathcal{Q}^{jkl}_{\acute{b}m} + \frac{1}{\sqrt{15}}\overline{\mathcal{Q}}_{\acute{a}kl}\gamma^A\mathcal{Q}^{jkl}_{\acute{b}i} + \frac{1}{\sqrt{15}}\overline{\mathcal{Q}}^{j}_{\acute{a}ikl}\gamma^A\mathcal{Q}^{kl}_{\acute{b}} + \frac{17}{15\sqrt{2}}\overline{\mathcal{Q}}_{\acute{a}ki}\gamma^A\mathcal{Q}^{jk}_{\acute{b}} \\
& -\sqrt{\frac{3}{10}}\overline{\mathcal{Q}}^{(S)}_{\acute{a}ki}\gamma^A\mathcal{Q}^{jk}_{\acute{b}} + \sqrt{\frac{3}{10}}\overline{\mathcal{Q}}_{\acute{a}ki}\gamma^A\mathcal{Q}^{jk}_{(S)\acute{b}} - \frac{1}{\sqrt{2}}\overline{\mathcal{Q}}^{(S)}_{\acute{a}ki}\gamma^A\mathcal{Q}^{jk}_{(S)\acute{b}} - \sqrt{2}\overline{\mathcal{Q}}^{k}_{\acute{a}i}\gamma^A\mathcal{Q}^{j}_{\acute{b}k}\Bigg]\mathsf{G}^{i}_{Aj}\Bigg\}
\end{aligned} \tag{B.8}$$

C. Higgs sector quartic couplings

We discuss now the quartic couplings involving four vector-spinors. We will discuss specifically the quartic couplings that arise from the cubic couplings discussed in section 4 by elimination of the 1, 45 and 210 fields assuming they are heavy in the $144\times\overline{144}$ couplings and by elimination of 10, 120 and $126+\overline{126}$ assuming they are heavy for the 144×144 couplings. We first discuss the quartic couplings that arise from the elimination of 1, 10, 45. In this case we start with the superpotenial

$$\begin{aligned}
\mathsf{W}^{(1,45,210)} = {} & h^{(1)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu} > k^{(1)}_{\mathcal{X}}\Phi_{\mathcal{X}} + \frac{1}{2}\Phi_{\mathcal{X}}\mathcal{M}^{(1)}_{\mathcal{X}\mathcal{X}'}\Phi_{\mathcal{X}'} \\
& +\frac{1}{2!}h^{(45)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Sigma_{\rho\sigma}|\Upsilon_{(+)\acute{b}\mu} > k^{(45)}_{\mathcal{Y}}\Phi_{\rho\sigma\mathcal{Y}} + \frac{1}{2}\Phi_{\rho\sigma\mathcal{Y}}\mathcal{M}^{(45)}_{\mathcal{Y}\mathcal{Y}'}\Phi_{\rho\sigma\mathcal{Y}'} \\
& +\frac{1}{4!}h^{(210)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda]}|\Upsilon_{(+)\acute{b}\mu} > k^{(210)}_{\mathcal{Z}}\Phi_{\nu\rho\sigma\lambda\mathcal{Z}} \\
& +\frac{1}{2}\Phi_{\nu\rho\sigma\lambda\mathcal{Z}}\mathcal{M}^{(210)}_{\mathcal{Z}\mathcal{Z}'}\Phi_{\nu\rho\sigma\lambda\mathcal{Z}'}
\end{aligned} \tag{C.1}$$

We then eliminate $\Phi_{\mathcal{X}}$, $\Phi_{\rho\sigma\mathcal{Y}}$, $\Phi_{\nu\rho\sigma\lambda\mathcal{Z}}$ assuming they are superheavy using the F-flatness conditions

$$\frac{\partial \mathsf{W}^{(1,45,210)}}{\partial \Phi_{\mathcal{X}}}=0,\quad \frac{\partial \mathsf{W}^{(1,45,210)}}{\partial \Phi_{\rho\sigma\mathcal{Y}}}=0,\quad \frac{\partial \mathsf{W}^{(1,45,210)}}{\partial \Phi_{\nu\rho\sigma\lambda\mathcal{Z}}}=0 \tag{C.2}$$

We discuss now the individual contribution arising from the elimination of 1, 45 and 210 separately.

C.1 The $\left(144\times\overline{144}\right)_1\left(144\times\overline{144}\right)_1$ couplings

The $\left(144\times\overline{144}\right)_1\left(144\times\overline{144}\right)_1$ couplings gotten by the singlet mediation are given by

$$\mathsf{W}^{(1)}_{dim-5}=2\lambda^{^{(1)}}_{\acute{a}\acute{b},\acute{c}\acute{d}}<\Upsilon^*_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^*_{(-)\acute{c}\lambda}|B|\Upsilon_{(+)\acute{d}\lambda}> \tag{C.3}$$

where

$$\begin{aligned}<\Upsilon^*_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu}>=i\Bigg\{&\frac{3}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}+\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}+\frac{1}{10}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}+\frac{1}{2}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij}\\&+\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}-\frac{1}{6}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}ijk}-\frac{1}{2}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\Bigg\}\end{aligned} \tag{C.4}$$

Explicit evaluation of the above quantities gives

$$\begin{aligned}\mathsf{W}^{(1)}_{dim-5}=\lambda^{^{(1)}}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Bigg\{&-\frac{18}{25}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{\mathbf{T}}_{\acute{c}j}\mathcal{P}^{j}_{\acute{d}}-\frac{12}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}j}-\frac{6}{25}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}jk}\\&-\frac{6}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}jk}-\frac{12}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}j}+\frac{2}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{c}m}\mathcal{P}^{m}_{\acute{d}jkl}\\&+\frac{6}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}kl}\mathcal{P}^{kl}_{\acute{d}j}-2\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}j}-\frac{2}{5}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\mathcal{Q}^{jk\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}jk}\\&-2\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}jk}-4\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}j}+\frac{2}{3}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\mathcal{Q}^{klm\mathbf{T}}_{\acute{c}j}\mathcal{P}^{j}_{\acute{d}klm}\\&+2\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\mathcal{Q}^{j\mathbf{T}}_{\acute{c}kl}\mathcal{P}^{kl}_{\acute{d}j}-\frac{1}{50}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\mathcal{Q}^{kl\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}kl}-\frac{1}{5}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\mathcal{Q}^{kl\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}kl}\\&-\frac{2}{5}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}l}\mathcal{P}^{l}_{\acute{d}k}+\frac{1}{15}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\mathcal{Q}^{klm\mathbf{T}}_{\acute{c}n}\mathcal{P}^{n}_{\acute{d}klm}+\frac{1}{5}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}lm}\mathcal{P}^{lm}_{\acute{d}k}\\&-\frac{1}{2}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij}\mathcal{Q}^{kl\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}kl}-2\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{d}ij}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}l}\mathcal{P}^{l}_{\acute{d}k}+\frac{1}{3}\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{d}ij}\mathcal{Q}^{klm\mathbf{T}}_{\acute{c}n}\mathcal{P}^{n}_{\acute{d}klm}\\&+\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{d}ij}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}lm}\mathcal{P}^{lm}_{\acute{d}k}-2\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}l}\mathcal{P}^{l}_{\acute{d}k}+\frac{2}{3}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}\mathcal{Q}^{lmn\mathbf{T}}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}lmn}\\&+2\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}lm}\mathcal{P}^{lm}_{\acute{d}k}-\frac{1}{18}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}ijk}\mathcal{Q}^{mno\mathbf{T}}_{\acute{c}p}\mathcal{P}^{p}_{\acute{d}mno}-\frac{1}{3}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}ijk}\mathcal{Q}^{m\mathbf{T}}_{\acute{c}no}\mathcal{P}^{no}_{\acute{d}m}\\&-\frac{1}{2}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\mathcal{Q}^{l\mathbf{T}}_{\acute{c}mn}\mathcal{P}^{l}_{\acute{d}mn}\Bigg\}\end{aligned} \tag{C.5}$$

where we have defined

$$\lambda^{^{(1)}}_{\acute{a}\acute{b},\acute{c}\acute{d}}=h^{^{(1)}}_{\acute{a}\acute{b}}h^{^{(1)}}_{\acute{c}\acute{d}}k^{^{(1)}}_{\mathcal{X}}\left[\widetilde{\mathcal{M}}^{^{(1)}}\left\{\mathcal{M}^{^{(1)}}\widetilde{\mathcal{M}}^{^{(1)}}-\mathbf{1}\right\}\right]_{\mathcal{X}\mathcal{X}'}k^{^{(1)}}_{\mathcal{X}'} \tag{C.6}$$

Here and in the rest of the paper, we define

$$\widetilde{\mathcal{M}}^{(k)} = \left[\mathcal{M}^{(k)} + \left(\mathcal{M}^{(k)}\right)^{\mathbf{T}}\right]^{-1} \tag{C.7}$$

where (k) denotes the specific tensor representation.

C.2 The $\left(\mathbf{144}\times\overline{\mathbf{144}}\right)_{\mathbf{45}}\left(\mathbf{144}\times\overline{\mathbf{144}}\right)_{\mathbf{45}}$ couplings

The $\left(144\times\overline{144}\right)_{45}\left(144\times\overline{144}\right)_{45}$ couplings gotten by the 45 plet mediation are given by

$$\begin{aligned}\mathsf{W}^{(45)}_{dim-5} = \lambda^{(45)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[&-4 < \Upsilon^{*}_{(-)\acute{a}\mu}|Bb_ib_j|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^{*}_{(-)\acute{c}\lambda}|Bb_i^{\dagger}b_j^{\dagger}|\Upsilon_{(+)\acute{d}\lambda} >\\ &+4 < \Upsilon^{*}_{(-)\acute{a}\mu}|Bb_i^{\dagger}b_j|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^{*}_{(-)\acute{c}\lambda}|Bb_j^{\dagger}b_i|\Upsilon_{(+)\acute{d}\lambda} >\\ &-4 < \Upsilon^{*}_{(-)\acute{a}\mu}|Bb_n^{\dagger}b_n|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^{*}_{(-)\acute{c}\lambda}|B|\Upsilon_{(+)\acute{d}\lambda} >\\ &+5 < \Upsilon^{*}_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^{*}_{(-)\acute{c}\lambda}|B|\Upsilon_{(+)\acute{d}\lambda} >\Big]\end{aligned} \tag{C.8}$$

where the explicit evaluation in $SU(5)\times U(1)$ decomposition gives

$$\begin{aligned}< \Upsilon^{*}_{(-)\acute{a}\mu}|Bb_ib_j|\Upsilon_{(+)\acute{b}\mu} >= i\Big\{&-\mathcal{Q}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{ij}_{\acute{b}k} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{i}_{\acute{b}} + \sqrt{\frac{2}{15}}\mathcal{Q}^{ik\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}k}\\ &-\sqrt{\frac{2}{15}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}^{i}_{\acute{b}k} + \mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}k} + \frac{7}{10\sqrt{6}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}_{\acute{b}lm}\\ &-\frac{1}{3\sqrt{5}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}klm} + \frac{1}{2}\sqrt{\frac{3}{10}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}kl}\mathcal{P}_{\acute{b}mn}\\ &+\frac{1}{2\sqrt{2}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}kl}\mathcal{P}^{(S)}_{\acute{b}mn}\Big\}\end{aligned} \tag{C.9}$$

$$\begin{aligned}< \Upsilon^{*}_{(-)\acute{a}\mu}|Bb_i^{\dagger}b_j^{\dagger}|\Upsilon_{(+)\acute{b}\mu} >= i\Big\{&-\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}_{\acute{b}k} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{P}_{i\acute{b}} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}j} + \sqrt{\frac{2}{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}kj}\\ &-\sqrt{\frac{2}{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}j}\mathcal{P}_{\acute{b}ki} + \mathcal{Q}^{k\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}ijk} + \frac{7}{10\sqrt{6}}\epsilon_{ijklm}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}^{m}_{\acute{b}}\\ &-\frac{1}{3\sqrt{5}}\epsilon_{ijklm}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}} + \frac{1}{2}\sqrt{\frac{3}{10}}\epsilon_{ijklm}\mathcal{Q}^{kn\mathbf{T}}_{\acute{a}}\mathcal{P}^{lm}_{\acute{b}n}\\ &+\frac{1}{2\sqrt{2}}\epsilon_{ijklm}\mathcal{Q}^{kn\mathbf{T}}_{\acute{a}(S)}\mathcal{P}^{lm}_{\acute{b}n}\Big\}\end{aligned} \tag{C.10}$$

$$\begin{aligned}< \Upsilon^{*}_{(-)\acute{a}\mu}|Bb_i^{\dagger}b_j|\Upsilon_{(+)\acute{b}\mu} >= i\Big\{&\mathcal{Q}^{l\mathbf{T}}_{\acute{a}ik}\mathcal{P}^{kj}_{\acute{b}l} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}ik}\mathcal{P}^{k}_{\acute{b}} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}^{kj}_{\acute{b}i}\\ &-\frac{3}{20}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{j}_{\acute{b}} - \frac{1}{20}\delta^{j}_{i}\mathcal{Q}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}} + \frac{1}{2}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}m}\mathcal{P}^{m}_{\acute{b}ikl}\\ &-\frac{1}{6}\delta^{j}_{i}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}klm} - \frac{1}{\sqrt{30}}\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}i}\mathcal{P}_{kl\acute{b}} - \frac{1}{\sqrt{30}}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}ikl}\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{30}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ki}+\frac{1}{6}\delta^j_i\,\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}kl}-\frac{1}{2}\sqrt{\frac{3}{5}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ki}\\
&+\frac{1}{2}\sqrt{\frac{3}{5}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{a}}\mathcal{P}_{\acute{b}ki}-\frac{1}{2}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ki}+\frac{1}{2}\delta^j_i\,\mathcal{Q}^{kl\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}kl}\\
&\qquad\qquad\qquad -\mathcal{Q}^{j\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}i}+\delta^j_i\,\mathcal{Q}^{l\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}l}\Big\}
\end{aligned}
\tag{C.11}
$$

$$
\begin{aligned}
<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{n}b_n|\Upsilon_{(+)\acute{b}\mu}>=i\Big\{&4\mathcal{Q}^{i\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}+\frac{4}{5}\mathcal{Q}^{ij\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}+2\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij}-\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{ij}_{\acute{b}k}\\
&-\frac{2}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{i}_{\acute{b}}-\frac{1}{3}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}ijk}\Big\}
\end{aligned}
\tag{C.12}
$$

where

$$
\lambda^{(45)}_{\acute{a}\acute{b},\acute{c}\acute{d}}=h^{(45)}_{\acute{a}\acute{b}}\,h^{(45)}_{\acute{c}\acute{d}}\,k^{(45)}_{\mathcal{Y}}\left[\widetilde{\mathcal{M}}^{(45)}\left\{\mathcal{M}^{(45)}\widetilde{\mathcal{M}}^{(45)}-\mathbf{1}\right\}\right]_{\mathcal{Y}\mathcal{Y}'}k^{(45)}_{\mathcal{Y}'}
\tag{C.13}
$$

C.3 The $\left(\mathbf{144}\times\overline{\mathbf{144}}\right)_{\mathbf{210}}\left(\mathbf{144}\times\overline{\mathbf{144}}\right)_{\mathbf{210}}$ couplings

The $\left(144\times\overline{144}\right)_{210}\left(144\times\overline{144}\right)_{210}$ couplings are gotten by 210 mediation and are given by

$$
\begin{aligned}
\mathsf{W}^{(210)}_{dim-5}=-\frac{1}{18}\lambda^{(210)}_{ab,cd}\Big[&8<\Upsilon^{*}_{(-)\acute{a}\nu}|Bb^{\dagger}_{i}b_jb_kb_l|\Upsilon_{(+)\acute{b}\nu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{j}b^{\dagger}_{k}b^{\dagger}_{l}b_i|\Upsilon_{(+)\acute{d}\lambda}>\\
&-6<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{i}b^{\dagger}_{j}b_kb_l|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{k}b^{\dagger}_{l}b_ib_j|\Upsilon_{(+)\acute{d}\lambda}>\\
&-2<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb_ib_jb_kb_l|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{i}b^{\dagger}_{j}b^{\dagger}_{k}b^{\dagger}_{l}|\Upsilon_{(+)\acute{d}\lambda}>\\
&+24<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{i}b_j|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{j}b^{\dagger}_{n}b_nb_i|\Upsilon_{(+)\acute{d}\lambda}>\\
&-12<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{i}b^{\dagger}_{j}|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{n}b_nb_ib_j|\Upsilon_{(+)\acute{d}\lambda}>\\
&-12<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb_ib_j|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{i}b^{\dagger}_{j}b^{\dagger}_{n}b_n|\Upsilon_{(+)\acute{d}\lambda}>\\
&-6<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{m}b_m|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{n}b_n|\Upsilon_{(+)\acute{d}\lambda}>\\
&-6<\Upsilon^{*}_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{m}b^{\dagger}_{n}b_nb_m|\Upsilon_{(+)\acute{d}\lambda}>\\
&+18<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb_ib_j|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{i}b^{\dagger}_{j}|\Upsilon_{(+)\acute{d}\lambda}>\\
&-18<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{i}b_j|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{j}b_i|\Upsilon_{(+)\acute{d}\lambda}>\\
&+24<\Upsilon^{*}_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{n}b_n|\Upsilon_{(+)\acute{d}\lambda}>\\
&-15<\Upsilon^{*}_{(-)\acute{a}\mu}|B|\Upsilon_{(+)\acute{b}\mu}><\Upsilon^{*}_{(-)\acute{c}\lambda}|B|\Upsilon_{(+)\acute{d}\lambda}>\Big]
\end{aligned}
\tag{C.14}
$$

We carry out now an $SU(5)\times U(1)$ decomposition of these and get

$$
\begin{aligned}
<\Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{i}b_jb_kb_l|\Upsilon_{(+)\acute{b}\mu}>=i\Big\{&\sqrt{\frac{2}{15}}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}i}-\sqrt{\frac{2}{15}}\left(\delta^k_i\,\mathcal{Q}^{nl\mathbf{T}}_{\acute{a}}-\delta^l_i\,\mathcal{Q}^{nk\mathbf{T}}_{\acute{a}}\right)\mathcal{P}^{j}_{\acute{b}n}\\
&+\sqrt{\frac{2}{15}}\mathcal{Q}^{lj\mathbf{T}}_{\acute{a}}\mathcal{P}^{k}_{\acute{b}i}-\sqrt{\frac{2}{15}}\left(\delta^l_i\,\mathcal{Q}^{nj\mathbf{T}}_{\acute{a}}-\delta^j_i\,\mathcal{Q}^{nl\mathbf{T}}_{\acute{a}}\right)\mathcal{P}^{k}_{\acute{b}n}\\
&+\sqrt{\frac{2}{15}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{a}}\mathcal{P}^{l}_{\acute{b}i}-\sqrt{\frac{2}{15}}\left(\delta^j_i\,\mathcal{Q}^{nk\mathbf{T}}_{\acute{a}}-\delta^k_i\,\mathcal{Q}^{nj\mathbf{T}}_{\acute{a}}\right)\mathcal{P}^{l}_{\acute{b}n}
\end{aligned}
$$

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$$
\begin{aligned}
&-\mathcal{Q}^{jkl\mathbf{T}}_{\acute{a}m}\mathcal{P}^{m}_{\acute{b}i} + \left(\delta^{j}_{i}\,\mathcal{Q}^{nkl\mathbf{T}}_{\acute{a}m} + \delta^{k}_{i}\,\mathcal{Q}^{nlj\mathbf{T}}_{\acute{a}m} + \delta^{l}_{i}\,\mathcal{Q}^{njk\mathbf{T}}_{\acute{a}m}\right)\mathcal{P}^{m}_{\acute{b}n} \\
&-\sqrt{\frac{3}{10}}\epsilon^{jklmn}\,\mathcal{Q}^{p\mathbf{T}}_{\acute{a}mi}\mathcal{P}_{\acute{b}np} - \frac{1}{\sqrt{2}}\epsilon^{jklmn}\,\mathcal{Q}^{p\mathbf{T}}_{\acute{a}mi}\mathcal{P}^{(S)}_{\acute{b}np} \\
&+\frac{1}{10}\sqrt{\frac{3}{2}}\epsilon^{jklmn}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}mn} + \frac{1}{10}\sqrt{\frac{3}{2}}\epsilon^{jklmn}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}m}\mathcal{P}_{\acute{b}ni} \\
&+\frac{1}{2\sqrt{10}}\epsilon^{jklmn}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}m}\mathcal{P}^{(S)}_{\acute{b}ni}\Bigg\} \qquad \text{(C.15)}
\end{aligned}
$$

$$
\begin{aligned}
< \Upsilon^{*}_{(-)\acute{c}\lambda}|Bb^{\dagger}_{j}b^{\dagger}_{k}b^{\dagger}_{l}b_{i}|\Upsilon_{(+)\acute{d}\lambda} >= i\Bigg\{&\sqrt{\frac{2}{15}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}j}\mathcal{P}_{\acute{d}kl} - \sqrt{\frac{2}{15}}\mathcal{Q}^{m\mathbf{T}}_{\acute{c}j}\left(\delta^{i}_{k}\mathcal{P}_{\acute{d}ml} - \delta^{i}_{l}\mathcal{P}_{\acute{d}mk}\right) \\
&+\sqrt{\frac{2}{15}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}l}\mathcal{P}_{\acute{d}jk} - \sqrt{\frac{2}{15}}\mathcal{Q}^{m\mathbf{T}}_{\acute{c}l}\left(\delta^{i}_{j}\mathcal{P}_{\acute{d}mk} - \delta^{i}_{k}\mathcal{P}_{\acute{d}mj}\right) \\
&+\sqrt{\frac{2}{15}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}k}\mathcal{P}_{\acute{d}lj} - \sqrt{\frac{2}{15}}\mathcal{Q}^{m\mathbf{T}}_{\acute{c}k}\left(\delta^{i}_{l}\mathcal{P}_{\acute{d}mj} - \delta^{i}_{j}\mathcal{P}_{\acute{d}ml}\right) \\
&-\mathcal{Q}^{i\mathbf{T}}_{\acute{c}m}\mathcal{P}^{m}_{\acute{d}jkl} + \mathcal{Q}^{m\mathbf{T}}_{\acute{c}n}\left(\delta^{i}_{j}\mathcal{P}^{n}_{\acute{d}mkl} + \delta^{i}_{k}\mathcal{P}^{n}_{\acute{d}mlj} + \delta^{i}_{l}\mathcal{P}^{n}_{\acute{d}mjk}\right) \\
&+\sqrt{\frac{3}{10}}\epsilon_{jklmn}\,\mathcal{Q}^{mp\mathbf{T}}_{\acute{c}}\mathcal{P}^{ni}_{\acute{d}p} + \frac{1}{\sqrt{2}}\epsilon_{jklmn}\,\mathcal{Q}^{mp\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{ni}_{\acute{d}p} \\
&+\frac{1}{10}\sqrt{\frac{3}{2}}\epsilon_{jklmn}\,\mathcal{Q}^{mn\mathbf{T}}_{\acute{c}}\mathcal{P}^{i}_{\acute{d}} - \frac{1}{10}\sqrt{\frac{3}{2}}\epsilon_{jklmn}\,\mathcal{Q}^{mi\mathbf{T}}_{\acute{c}}\mathcal{P}^{n}_{\acute{d}} \\
&-\frac{1}{2\sqrt{10}}\epsilon_{jklmn}\,\mathcal{Q}^{mi\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{n}_{\acute{d}}\Bigg\} \qquad \text{(C.16)}
\end{aligned}
$$

$$
\begin{aligned}
< \Upsilon^{*}_{(-)\acute{a}\mu}|Bb^{\dagger}_{i}b^{\dagger}_{j}b_{k}b_{l}|\Upsilon_{(+)\acute{b}\mu} >= i\Bigg\{&\mathcal{Q}^{m\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{kl}_{\acute{b}m} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{l}_{\acute{b}} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{l\mathbf{T}}_{\acute{a}ij}\mathcal{P}^{k}_{\acute{b}} \\
&+\frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{P}^{kl}_{\acute{b}i} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{P}^{kl}_{\acute{b}j} - \frac{1}{20}\left(\delta^{k}_{j}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}i} - \delta^{k}_{i}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\right)\mathcal{P}^{l}_{\acute{b}} \\
&-\frac{1}{20}\left(\delta^{l}_{i}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}j} - \delta^{l}_{j}\,\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\right)\mathcal{P}^{k}_{\acute{b}} + \mathcal{Q}^{klm\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}ijm} \\
&-\frac{1}{2}\mathcal{Q}^{kmn\mathbf{T}}_{\acute{a}p}\left(\delta^{l}_{j}\mathcal{P}^{p}_{\acute{b}imn} - \delta^{l}_{i}\mathcal{P}^{p}_{\acute{b}jmn}\right) - \frac{1}{2}\mathcal{Q}^{lmn\mathbf{T}}_{\acute{a}p}\left(\delta^{k}_{i}\mathcal{P}^{p}_{\acute{b}jmn} - \delta^{k}_{j}\mathcal{P}^{p}_{\acute{b}imn}\right) \\
&+\left(\delta^{l}_{i}\delta^{k}_{j} - \delta^{k}_{i}\delta^{l}_{j}\right)\left(-\frac{1}{6}\mathcal{Q}^{mnp\mathbf{T}}_{\acute{a}q}\mathcal{P}^{q}_{\acute{b}mnp} + \frac{7}{30}\mathcal{Q}^{mn\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}mn} + \mathcal{Q}^{m\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}m} + \frac{1}{2}\mathcal{Q}^{mn\mathbf{T}}_{(S)\acute{a}}\mathcal{P}_{(S)\acute{b}mn}\right) \\
&+\sqrt{\frac{2}{15}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}mj} - \sqrt{\frac{2}{15}}\mathcal{Q}^{klm\mathbf{T}}_{\acute{a}j}\mathcal{P}_{\acute{b}mi} \\
&-\frac{1}{\sqrt{30}}\left(\delta^{k}_{j}\,\mathcal{Q}^{lmn\mathbf{T}}_{\acute{a}i} - \delta^{k}_{i}\,\mathcal{Q}^{lmn\mathbf{T}}_{\acute{a}j} - \delta^{l}_{j}\,\mathcal{Q}^{kmn\mathbf{T}}_{\acute{a}i} + \delta^{l}_{i}\,\mathcal{Q}^{kmn\mathbf{T}}_{\acute{a}j}\right)\mathcal{P}_{\acute{b}mn} \\
&-\sqrt{\frac{2}{15}}\mathcal{Q}^{lm\mathbf{T}}_{\acute{a}}\mathcal{P}^{k}_{\acute{b}mij} + \sqrt{\frac{2}{15}}\mathcal{Q}^{km\mathbf{T}}_{\acute{a}}\mathcal{P}^{l}_{\acute{b}mij} \\
&-\frac{1}{\sqrt{30}}\mathcal{Q}^{mn\mathbf{T}}_{\acute{a}}\left(\delta^{l}_{i}\mathcal{P}^{k}_{\acute{b}jmn} - \delta^{l}_{j}\mathcal{P}^{k}_{\acute{b}imn} - \delta^{k}_{i}\mathcal{P}^{l}_{\acute{b}jmn} + \delta^{k}_{j}\mathcal{P}^{l}_{\acute{b}imn}\right)
\end{aligned}
$$

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$$
\begin{aligned}
&+\frac{1}{2}\left(\delta^l_i \mathcal{Q}^{km\mathbf{T}}_{\acute{a}} - \delta^k_i \mathcal{Q}^{lm\mathbf{T}}_{\acute{a}}\right)\left(\frac{1}{3}\mathcal{P}_{\acute{b}mj} - \sqrt{\frac{3}{5}}\mathcal{P}_{(S)\acute{b}mj}\right)\\
&+\frac{1}{2}\left(\delta^k_j \mathcal{Q}^{lm\mathbf{T}}_{\acute{a}} - \delta^l_j \mathcal{Q}^{km\mathbf{T}}_{\acute{a}}\right)\left(\frac{1}{3}\mathcal{P}_{\acute{b}mi} - \sqrt{\frac{3}{5}}\mathcal{P}_{(S)\acute{b}mi}\right)\\
&+\frac{1}{2}\left(\delta^l_i \mathcal{Q}^{km\mathbf{T}}_{(S)\acute{a}} - \delta^k_i \mathcal{Q}^{lm\mathbf{T}}_{(S)\acute{a}}\right)\left(\sqrt{\frac{3}{5}}\mathcal{P}_{\acute{b}mj} - \mathcal{P}_{(S)\acute{b}mj}\right)\\
&+\frac{1}{2}\left(\delta^k_j \mathcal{Q}^{lm\mathbf{T}}_{(S)\acute{a}} - \delta^l_j \mathcal{Q}^{km\mathbf{T}}_{(S)\acute{a}}\right)\left(\sqrt{\frac{3}{5}}\mathcal{P}_{\acute{b}mi} - \mathcal{P}_{(S)\acute{b}mi}\right)\\
&-\left(\delta^k_j \mathcal{Q}^{l\mathbf{T}}_{\acute{a}m} - \delta^l_j \mathcal{Q}^{k\mathbf{T}}_{\acute{a}m}\right)\mathcal{P}^m_{\acute{b}i} - \left(\delta^l_i \mathcal{Q}^{k\mathbf{T}}_{\acute{a}m} - \delta^k_i \mathcal{Q}^{l\mathbf{T}}_{\acute{a}m}\right)\mathcal{P}^m_{\acute{b}j}\\
&+\frac{2}{15}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij}\Big\} \qquad \text{(C.17)}
\end{aligned}
$$

$$
\begin{aligned}
< \Upsilon^*_{(-)\acute{a}\mu}|Bb_ib_jb_kb_l|\Upsilon_{(+)\acute{b}\mu} >= i\Big\{ &-\sqrt{\frac{3}{10}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}nm} + \frac{1}{\sqrt{2}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}nm}\\
&+\frac{2}{\sqrt{5}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{a}n}\mathcal{P}^n_{\acute{b}m}\Big\} \qquad \text{(C.18)}
\end{aligned}
$$

$$
\begin{aligned}
< \Upsilon^*_{(-)\acute{c}\lambda}|Bb^\dagger_ib^\dagger_jb^\dagger_kb^\dagger_l|\Upsilon_{(+)\acute{d}\lambda} >= i\Big\{ &\sqrt{\frac{3}{10}}\epsilon_{ijklm}\mathcal{Q}^{mn\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}n} + \frac{1}{\sqrt{2}}\epsilon_{ijklm}\mathcal{Q}^{mn\mathbf{T}}_{(S)\acute{c}}\mathcal{P}_{\acute{d}n}\\
&+\frac{2}{\sqrt{5}}\epsilon_{ijklm}\mathcal{Q}^{m\mathbf{T}}_{\acute{c}n}\mathcal{P}^n_{\acute{d}}\Big\} \qquad \text{(C.19)}
\end{aligned}
$$

$$
\begin{aligned}
< \Upsilon^*_{(-)\acute{c}\lambda}|Bb^\dagger_jb^\dagger_nb_nb_i|\Upsilon_{(+)\acute{d}\lambda} >= i\Big\{ &\mathcal{Q}^{l\mathbf{T}}_{\acute{c}jk}\mathcal{P}^{ki}_{\acute{d}l} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{i\mathbf{T}}_{\acute{c}jk}\mathcal{P}^k_{\acute{d}} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{c}k}\mathcal{P}^{ki}_{\acute{d}j}\\
&-\frac{3}{20}\mathcal{Q}^{\mathbf{T}}_{\acute{c}j}\mathcal{P}^i_{\acute{d}} - \frac{1}{20}\delta^i_j\mathcal{Q}^{\mathbf{T}}_{\acute{c}k}\mathcal{P}^k_{\acute{d}} + \frac{1}{2}\mathcal{Q}^{ikl\mathbf{T}}_{\acute{c}m}\mathcal{P}^m_{\acute{d}jkl}\\
&-\frac{1}{6}\delta^i_j\mathcal{Q}^{klm\mathbf{T}}_{\acute{c}n}\mathcal{P}^n_{\acute{d}klm} - \frac{1}{\sqrt{30}}\mathcal{Q}^{ikl\mathbf{T}}_{\acute{c}j}\mathcal{P}^{kl}_{\acute{d}} - \frac{1}{\sqrt{30}}\mathcal{Q}^{kl\mathbf{T}}_{\acute{c}}\mathcal{P}^i_{\acute{d}jkl}\\
&+\frac{19}{30}\mathcal{Q}^{ik\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}kj} + \frac{23}{30}\delta^i_j\mathcal{Q}^{kl\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}kl} - \frac{3}{2}\sqrt{\frac{3}{5}}\mathcal{Q}^{ik\mathbf{T}}_{\acute{c}}\mathcal{P}^{(S)}_{\acute{d}kj}\\
&+\frac{3}{2}\sqrt{\frac{3}{5}}\mathcal{Q}^{ik\mathbf{T}}_{(S)\acute{c}}\mathcal{P}_{\acute{d}kj} - \frac{3}{2}\mathcal{Q}^{ik\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}kj} + \frac{3}{2}\delta^i_j\mathcal{Q}^{kl\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}kl}\\
&-3\mathcal{Q}^{i\mathbf{T}}_{\acute{c}k}\mathcal{P}^k_{\acute{d}j} + 3\delta^i_j\mathcal{Q}^{l\mathbf{T}}_{\acute{c}k}\mathcal{P}^k_{\acute{d}l}\Big\} \qquad \text{(C.20)}
\end{aligned}
$$

$$
< \Upsilon^*_{(-)\acute{c}\lambda}|Bb^\dagger_nb_nb_ib_j|\Upsilon_{(+)\acute{d}\lambda} >= i\Big\{ \sqrt{\frac{2}{15}}\mathcal{Q}^{ik\mathbf{T}}_{\acute{c}}\mathcal{P}^j_{\acute{d}k} - \sqrt{\frac{2}{15}}\mathcal{Q}^{jk\mathbf{T}}_{\acute{c}}\mathcal{P}^i_{\acute{d}k} + \mathcal{Q}^{ijk\mathbf{T}}_{\acute{c}l}\mathcal{P}^l_{\acute{d}k}
$$

$$+\frac{1}{10}\sqrt{\frac{3}{2}}\epsilon^{ijklm}\mathcal{Q}^{\mathbf{T}}_{\acute{c}k}\mathcal{P}_{\acute{d}lm}+\frac{1}{2}\sqrt{\frac{3}{10}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}kl}\mathcal{P}_{\acute{d}mn}$$
$$+\frac{1}{2\sqrt{2}}\epsilon^{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}kl}\mathcal{P}^{(S)}_{\acute{d}mn}\Bigg\} \quad \text{(C.21)}$$

$$< \Upsilon^*_{(-)\acute{c}\lambda}|Bb_i^\dagger b_j^\dagger b_n^\dagger b_n|\Upsilon_{(+)\acute{d}\lambda}>= i\Bigg\{\sqrt{\frac{2}{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}i}\mathcal{P}_{\acute{d}kj}-\sqrt{\frac{2}{15}}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}j}\mathcal{P}_{\acute{d}ki}+\mathcal{Q}^{k\mathbf{T}}_{\acute{c}l}\mathcal{P}^{l}_{\acute{d}ijk}$$
$$+\frac{1}{10}\sqrt{\frac{3}{2}}\epsilon_{ijklm}\mathcal{Q}^{kl\mathbf{T}}_{\acute{c}}\mathcal{P}^{m}_{\acute{d}}+\frac{1}{2}\sqrt{\frac{3}{10}}\epsilon_{ijklm}\mathcal{Q}^{kn\mathbf{T}}_{\acute{c}}\mathcal{P}^{lm}_{\acute{d}n}$$
$$+\frac{1}{2\sqrt{2}}\epsilon_{ijklm}\mathcal{Q}^{kn\mathbf{T}}_{\acute{d}(S)}\mathcal{P}^{lm}_{\acute{b}n}\Bigg\} \quad \text{(C.22)}$$

$$< \Upsilon^*_{(-)\acute{c}\lambda}|Bb_m^\dagger b_n^\dagger b_n b_m|\Upsilon_{(+)\acute{d}\lambda}>= i\Bigg\{12\mathcal{Q}^{i\mathbf{T}}_{\acute{c}j}\mathcal{P}^{j}_{\acute{d}i}+\frac{16}{5}\mathcal{Q}^{ij\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}ij}+6\mathcal{Q}^{ij\mathbf{T}}_{(S)\acute{c}}\mathcal{P}^{(S)}_{\acute{d}ij}$$
$$-\mathcal{Q}^{k\mathbf{T}}_{\acute{c}ij}\mathcal{P}^{ij}_{\acute{d}k}-\frac{2}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{c}i}\mathcal{P}^{i}_{\acute{d}}-\frac{1}{3}\mathcal{Q}^{ijk\mathbf{T}}_{\acute{c}l}\mathcal{P}^{l}_{\acute{d}ijk}\Bigg\} \quad \text{(C.23)}$$

where

$$\lambda^{(210)}_{\acute{a}\acute{b},\acute{c}\acute{d}}=h^{(210)}_{\acute{a}\acute{b}}h^{(210)}_{\acute{c}\acute{d}}k^{(210)}_{\mathcal{Z}}\left[\widetilde{\mathcal{M}}^{(210)}\left\{\mathcal{M}^{(210)}\widetilde{\mathcal{M}}^{(210)}-\mathbf{1}\right\}\right]_{\mathcal{Z}\mathcal{Z}'}k^{(210)}_{\mathcal{Z}'} \quad \text{(C.24)}$$

C.4 The $\left(\overline{\mathbf{144}}\times\overline{\mathbf{144}}\right)_{\mathbf{10}}\left(\overline{\mathbf{144}}\times\overline{\mathbf{144}}\right)_{\mathbf{10}}$ couplings

Here we consider the quartic interactions that arise from mediation by the 10 plet of Higgs. We begin by considering the superpotential

$$\mathsf{W}^{(10)'}=\frac{1}{2}\Phi_{\nu\mathcal{U}}\mathcal{M}^{(10)}_{\mathcal{U}\mathcal{U}'}\Phi_{\nu\mathcal{U}'}+h^{(10)}_{\acute{a}\acute{b}}< \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_\nu|\Upsilon_{(+)\acute{b}\mu}>k^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}}$$
$$+\bar{h}^{(10)}_{\acute{a}\acute{b}}< \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_\nu|\Upsilon_{(-)\acute{b}\mu}>\bar{k}^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}} \quad \text{(C.25)}$$

Elimination of the $\Phi_{\nu\mathcal{U}}$ as a superheavy field using the F-flatness condition

$$\frac{\partial\mathsf{W}^{(10)'}}{\partial\Phi_{\nu\mathcal{U}}}=0 \quad \text{(C.26)}$$

leads to the quartic interaction generated by 10 mediation.

$$\mathsf{W}^{(\overline{144}\times\overline{144})_{10}(\overline{144}\times\overline{144})_{10}}=2\lambda^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}}< \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_\rho|\Upsilon_{(+)\acute{b}\mu}>< \Upsilon^*_{(+)\acute{c}\nu}|B\Gamma_\rho|\Upsilon_{(+)\acute{d}\nu}>$$
$$=8\lambda^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}}< \Upsilon^*_{(+)\acute{a}\mu}|Bb_i|\Upsilon_{(+)\acute{b}\mu}>< \Upsilon^*_{(+)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(+)\acute{d}\nu}>$$
$$=4\lambda^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\left(8\mathbf{P}^{\mathbf{T}}_{\acute{a}i\mu}\mathbf{P}^{ij}_{\acute{b}\mu}\mathbf{P}^{\mathbf{T}}_{\acute{c}j\nu}\mathbf{P}_{\acute{d}\nu}-\epsilon_{jklmn}\mathbf{P}^{\mathbf{T}}_{\acute{a}i\mu}\mathbf{P}^{ij}_{\acute{b}\mu}\mathbf{P}^{kl\mathbf{T}}_{\acute{c}\nu}\mathbf{P}^{mn}_{\acute{d}\nu}\right) \quad \text{(C.27)}$$

where

$$\lambda^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}=h^{(10)(+)}_{\acute{a}\acute{b}}h^{(10)(+)}_{\acute{c}\acute{d}}k^{(10)}_{\mathcal{U}}\left[\widetilde{\mathcal{M}}^{(10)}\left\{\mathcal{M}^{(10)}\widetilde{\mathcal{M}}^{(10)}-\mathbf{1}\right\}\right]_{\mathcal{U}\mathcal{U}'}k^{(10)}_{\mathcal{U}'} \quad \text{(C.28)}$$

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C.5 The $(\mathbf{144}\times\mathbf{144})_{\mathbf{10}}$ $(\mathbf{144}\times\mathbf{144})_{\mathbf{10}}$ couplings

An analysis similar to the above gives in this case the following

$$
\begin{aligned}
\mathsf{W}^{(\overline{144}\times\overline{144})_{10}(\overline{144}\times\overline{144})_{10}} &= 2\bar{\lambda}^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_\rho|\Upsilon_{(-)\acute{b}\mu} >< \Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_\rho|\Upsilon_{(-)\acute{d}\nu} > \\
&= 8\bar{\lambda}^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Upsilon^*_{(-)\acute{a}\mu}|Bb_i|\Upsilon_{(-)\acute{b}\mu} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute{d}\nu} > \\
&= 4\bar{\lambda}^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \left(-8\mathbf{Q}^{i\mathbf{T}}_{\acute{a}\mu}\mathbf{Q}_{\acute{b}ij\mu}\mathbf{Q}^{j\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}_{\acute{d}\nu} + \epsilon^{jklmn}\mathbf{Q}^{i\mathbf{T}}_{\acute{a}\mu}\mathbf{Q}_{\acute{b}ij\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}kl\nu}\mathbf{Q}_{\acute{d}mn\nu}\right)
\end{aligned}
\tag{C.29}
$$

where

$$
\bar{\lambda}^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} = \bar{h}^{(10)(+)}_{\acute{a}\acute{b}}\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}\bar{k}^{(10)}_{\mathcal{U}}\left[\widetilde{\mathcal{M}}^{(10)}\left\{\mathcal{M}^{(10)}\widetilde{\mathcal{M}}^{(10)}-\mathbf{1}\right\}\right]_{\mathcal{U}\mathcal{U}'}\bar{k}^{(10)}_{\mathcal{U}'}
\tag{C.30}
$$

C.6 The $(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{10}}$ $(\mathbf{144}\times\mathbf{144})_{\mathbf{10}}$ couplings

An analysis similar to above gives

$$
\begin{aligned}
\mathsf{W}^{(\overline{144}\times\overline{144})_{10}(144\times144)_{10}} &= -2\theta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_\rho|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_\rho|\Upsilon_{(-)\acute{d}\nu} > \\
&= -4\theta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[< \Upsilon^*_{(+)\acute{a}\mu}|Bb_i|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute{d}\nu} > \\
&\quad + < \Upsilon^*_{(+)\acute{a}\mu}|Bb_i^\dagger|\Upsilon_{(+)\acute{b}\mu} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i|\Upsilon_{(-)\acute{d}\nu} >\Big] \\
&= 2\theta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big(8\mathbf{P}^{\mathbf{T}}_{\acute{a}i\nu}\mathbf{P}^{ij}_{\acute{b}\nu}\mathbf{Q}^{k\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}kj\mu} - 8\mathbf{P}^{\mathbf{T}}_{\acute{a}i\nu}\mathbf{P}_{\acute{b}\nu}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu} \\
&\quad -\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\nu}\mathbf{P}^{kl}_{\acute{b}\nu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}kl\mu}\mathbf{Q}_{\acute{d}ij\mu} + \mathbf{P}^{ij\mathbf{T}}_{\acute{a}\nu}\mathbf{P}^{kl}_{\acute{b}\nu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\mu}\mathbf{Q}_{\acute{d}jl\mu} \\
&\quad -\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\nu}\mathbf{P}^{kl}_{\acute{b}\nu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}il\mu}\mathbf{Q}_{\acute{d}jk\mu} + \epsilon^{ijklm}\mathbf{P}^{\mathbf{T}}_{\acute{a}i\nu}\mathbf{P}_{\acute{b}\nu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\mu}\mathbf{Q}_{\acute{d}lm\mu} \\
&\quad +\epsilon_{ijklm}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\nu}\mathbf{P}^{kl}_{\acute{b}\nu}\mathbf{Q}^{m\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu}\Big)
\end{aligned}
\tag{C.31}
$$

where

$$
\theta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} = h^{(10)(+)}_{\acute{a}\acute{b}}\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}k^{(10)}_{\mathcal{U}}\widetilde{\mathcal{M}}^{(10)}_{\mathcal{U}\mathcal{U}'}\bar{k}^{(10)}_{\mathcal{U}'}
\tag{C.32}
$$

Further details of the decomposition of the couplings generated by 10 mediation are given in appendix E.

C.7 The $(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{120}}$ $(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{120}}$ couplings

We begin by considering the superpotenial

$$
\begin{aligned}
\mathsf{W}^{(120)'} &= \frac{1}{2}\Phi_{\nu\rho\lambda V}\mathcal{M}^{(120)}_{VV'}\Phi_{\rho\nu\lambda V'} + \frac{1}{3!}h^{(120)}_{\acute{a}\acute{b}} < \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(+)\acute{b}\mu} > k^{(120)}_V\Phi_{\nu\rho\lambda V} \\
&\quad + \frac{1}{3!}\bar{h}^{(120)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(120)}_V\Phi_{\nu\rho\lambda V}
\end{aligned}
\tag{C.33}
$$

Eliminating $\Phi_{\nu\rho\lambda V}$ using the F flatness condition

$$
\frac{\partial\mathsf{W}^{(120)'}}{\partial\Phi_{\nu\rho\lambda V}} = 0
\tag{C.34}
$$

we obtain

$$
\begin{aligned}
\mathsf{W}^{(\overline{144}\times\overline{144})_{120}(\overline{144}\times\overline{144})_{120}} &= \frac{1}{18}\lambda^{(120)}_{\acute a\acute b,\acute c\acute d} < \Upsilon^*_{(+)\acute a\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(+)\acute d\nu}> \\
&= \frac{1}{18}\lambda^{(120)}_{\acute a\acute b,\acute c\acute d}\Big[< \Upsilon^*_{(+)\acute a\mu}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Upsilon_{(+)\acute d\nu}> \\
&\qquad -28 < \Upsilon^*_{(+)\acute a\mu}|B\Gamma_\nu|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|B\Gamma_\nu|\Upsilon_{(+)\acute d\nu}>\Big] \\
&= \frac{8}{9}\lambda^{(120)}_{\acute a\acute b,\acute c\acute d}\Big[< \Upsilon^*_{(+)\acute a\mu}|Bb_ib_jb_k|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|Bb_i^\dagger b_j^\dagger b_k^\dagger|\Upsilon_{(+)\acute d\nu}> \\
&\qquad -6 < \Upsilon^*_{(+)\acute a\mu}|Bb_i|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|Bb_i^\dagger|\Upsilon_{(+)\acute d\nu}> \\
&\qquad +3 < \Upsilon^*_{(+)\acute a\mu}|Bb_i^\dagger|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|Bb_n^\dagger b_n b_i|\Upsilon_{(+)\acute d\nu}> \\
&\qquad +3 < \Upsilon^*_{(+)\acute a\mu}|Bb_i|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|Bb_i^\dagger b_n^\dagger b_n|\Upsilon_{(+)\acute d\nu}> \\
&\qquad +3 < \Upsilon^*_{(+)\acute a\mu}|Bb_i^\dagger b_j^\dagger b_k|\Upsilon_{(+)\acute b\mu}>< \Upsilon^*_{(+)\acute c\nu}|Bb_k^\dagger b_i b_j|\Upsilon_{(+)\acute d\nu}>\Big] \\
&= \frac{8}{3}\lambda^{(120)(-)}_{\acute a\acute b,\acute c\acute d}\Big[-4\mathbf{P}^{\mathbf{T}}_{\acute a i\mu}\mathbf{P}_{\acute b j\mu}\mathbf{P}^{ij\mathbf{T}}_{\acute c\nu}\mathbf{P}_{\acute d\nu} + 4\mathbf{P}^{\mathbf{T}}_{\acute a\mu}\mathbf{P}_{\acute b i\mu}\mathbf{P}^{ij\mathbf{T}}_{\acute c\nu}\mathbf{P}_{\acute d j\nu} \\
&\qquad +\epsilon_{ijklm}\mathbf{P}^{ij\mathbf{T}}_{\acute a\mu}\mathbf{P}^{kn}_{\acute b\mu}\mathbf{P}^{\mathbf{T}}_{\acute c n\nu}\mathbf{P}^{lm}_{\acute d\nu}\Big]
\end{aligned}
\tag{C.35}
$$

where

$$
\lambda^{(120)(-)}_{\acute a\acute b,\acute c\acute d} = h^{(120)(-)}_{\acute a\acute b}\, h^{(120)(-)}_{\acute c\acute d}\, k^{(120)}_{V}\left[\widetilde{\mathcal{M}}^{(120)}\left\{\mathcal{M}^{(120)}\widetilde{\mathcal{M}}^{(120)} - \mathbf{1}\right\}\right]_{VV'} k^{(120)}_{V'} \tag{C.36}
$$

C.8 The $(\mathbf{144}\times\mathbf{144})_{\mathbf{120}}$ $(\mathbf{144}\times\mathbf{144})_{\mathbf{120}}$ couplings

An analysis similar to the above gives

$$
\begin{aligned}
\mathsf{W}^{(144\times144)_{120}(144\times144)_{120}} &= \frac{1}{18}\bar\lambda^{(120)}_{\acute a\acute b,\acute c\acute d} < \Upsilon^*_{(-)\acute a\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(-)\acute d\nu}> \\
&= \frac{1}{18}\bar\lambda^{(120)}_{\acute a\acute b,\acute c\acute d}\Big[< \Upsilon^*_{(-)\acute a\mu}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Upsilon_{(-)\acute d\nu}> \\
&\qquad -28 < \Upsilon^*_{(-)\acute a\mu}|B\Gamma_\nu|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|B\Gamma_\nu|\Upsilon_{(-)\acute d\nu}>\Big] \\
&= \frac{8}{9}\bar\lambda^{(120)}_{\acute a\acute b,\acute c\acute d}\Big[< \Upsilon^*_{(-)\acute a\mu}|Bb_ib_jb_k|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|Bb_i^\dagger b_j^\dagger b_k^\dagger|\Upsilon_{(-)\acute d\nu}> \\
&\qquad -6 < \Upsilon^*_{(-)\acute a\mu}|Bb_i|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute d\nu}> \\
&\qquad +3 < \Upsilon^*_{(-)\acute a\mu}|Bb_i^\dagger|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|Bb_n^\dagger b_n b_i|\Upsilon_{(-)\acute d\nu}> \\
&\qquad +3 < \Upsilon^*_{(-)\acute a\mu}|Bb_i|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|Bb_i^\dagger b_n^\dagger b_n|\Upsilon_{(-)\acute d\nu}> \\
&\qquad +3 < \Upsilon^*_{(-)\acute a\mu}|Bb_i^\dagger b_j^\dagger b_k|\Upsilon_{(-)\acute b\mu}>< \Upsilon^*_{(-)\acute c\nu}|Bb_k^\dagger b_i b_j|\Upsilon_{(-)\acute d\nu}>\Big] \\
&= \frac{8}{3}\bar\lambda^{(120)(-)}_{\acute a\acute b,\acute c\acute d}\Big[4\mathbf{Q}^{i\mathbf{T}}_{\acute a\mu}\mathbf{Q}^{j}_{\acute b\mu}\mathbf{Q}^{\mathbf{T}}_{\acute c ij\nu}\mathbf{Q}_{\acute d\nu} - 4\mathbf{Q}^{\mathbf{T}}_{\acute a\mu}\mathbf{Q}^{i}_{\acute b\mu}\mathbf{Q}^{\mathbf{T}}_{\acute c ij\nu}\mathbf{Q}^{j}_{\acute d\nu} \\
&\qquad -\epsilon^{ijklm}\mathbf{Q}^{\mathbf{T}}_{\acute a ij\mu}\mathbf{Q}_{\acute b kn\mu}\mathbf{Q}^{n\mathbf{T}}_{\acute c\nu}\mathbf{Q}_{\acute d lm\nu}\Big]
\end{aligned}
\tag{C.37}
$$

where

$$
\bar\lambda^{(120)(-)}_{\acute a\acute b,\acute c\acute d} = \bar h^{(120)(-)}_{\acute a\acute b}\, \bar h^{(120)(-)}_{\acute c\acute d}\, \bar k^{(120)}_{V}\left[\widetilde{\mathcal{M}}^{(120)}\left\{\mathcal{M}^{(120)}\widetilde{\mathcal{M}}^{(120)} - \mathbf{1}\right\}\right]_{VV'} \bar k^{(120)}_{V'} \tag{C.38}
$$

C.9 The $\left(\overline{\mathbf{144}}\times\overline{\mathbf{144}}\right)_{\mathbf{120}}(\mathbf{144}\times\mathbf{144})_{\mathbf{120}}$ couplings

Starting with cubic couplings involving the 120-plet of fields and following the same procedure as above one gets the following

$$
\begin{aligned}
\mathsf{W}^{(\overline{144}\times\overline{144})_{120}(144\times 144)_{120}} &= -\frac{1}{18}\theta^{(120)}_{\acute a\acute b,\acute c\acute d} < \Upsilon^{*}_{(+)\acute a\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\lambda]}|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\lambda]}|\Upsilon_{(-)\acute d\nu} > \\
&= -\frac{1}{18}\theta^{(120)}_{\acute a\acute b,\acute c\acute d}\Big[< \Upsilon^{*}_{(+)\acute a\mu}|B\Gamma_{\nu}\Gamma_{\rho}\Gamma_{\lambda}|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|B\Gamma_{\nu}\Gamma_{\rho}\Gamma_{\lambda}|\Upsilon_{(-)\acute d\nu} > \\
&\qquad -28 < \Upsilon^{*}_{(+)\acute a\mu}|B\Gamma_{\nu}|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|B\Gamma_{\nu}|\Upsilon_{(-)\acute d\nu} >\Big] \\
&= -\frac{4}{9}\theta^{(120)}_{\acute a\acute b,\acute c\acute d}\Big[< \Upsilon^{*}_{(+)\acute a\mu}|Bb_ib_jb_k|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|Bb^{\dagger}_ib^{\dagger}_jb^{\dagger}_k|\Upsilon_{(-)\acute d\nu} > \\
&\qquad -6 < \Upsilon^{*}_{(+)\acute a\mu}|Bb_i|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|Bb^{\dagger}_i|\Upsilon_{(-)\acute d\nu} > \\
&\qquad +3 < \Upsilon^{*}_{(+)\acute a\mu}|Bb^{\dagger}_i|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|Bb^{\dagger}_nb_nb_i|\Upsilon_{(-)\acute d\nu} > \\
&\qquad +3 < \Upsilon^{*}_{(+)\acute a\mu}|Bb_i|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|Bb^{\dagger}_ib^{\dagger}_nb_n|\Upsilon_{(-)\acute d\nu} > \\
&\qquad +3 < \Upsilon^{*}_{(+)\acute a\mu}|Bb^{\dagger}_ib^{\dagger}_jb_k|\Upsilon_{(+)\acute b\mu} >< \Upsilon^{*}_{(-)\acute c\nu}|Bb^{\dagger}_kb_ib_j|\Upsilon_{(-)\acute d\nu} > \\
&\qquad + < \Upsilon^{*}_{(-)\acute a\nu}|Bb_ib_jb_k|\Upsilon_{(-)\acute b\nu} >< \Upsilon^{*}_{(+)\acute c\mu}|Bb^{\dagger}_ib^{\dagger}_jb^{\dagger}_k|\Upsilon_{(+)\acute d\mu} > \\
&\qquad -6 < \Upsilon^{*}_{(-)\acute a\nu}|Bb_i|\Upsilon_{(-)\acute b\nu} >< \Upsilon^{*}_{(+)\acute c\mu}|Bb^{\dagger}_i|\Upsilon_{(+)\acute d\mu} > \\
&\qquad +3 < \Upsilon^{*}_{(-)\acute a\nu}|Bb^{\dagger}_i|\Upsilon_{(-)\acute b\nu} >< \Upsilon^{*}_{(+)\acute c\mu}|Bb^{\dagger}_nb_nb_i|\Upsilon_{(+)\acute d\mu} > \\
&\qquad +3 < \Upsilon^{*}_{(-)\acute a\nu}|Bb_i|\Upsilon_{(-)\acute b\nu} >< \Upsilon^{*}_{(+)\acute c\mu}|Bb^{\dagger}_ib^{\dagger}_nb_n|\Upsilon_{(+)\acute d\mu} > \\
&\qquad +3 < \Upsilon^{*}_{(-)\acute a\nu}|Bb^{\dagger}_ib^{\dagger}_jb_k|\Upsilon_{(-)\acute b\nu} >< \Upsilon^{*}_{(+)\acute c\mu}|Bb^{\dagger}_kb_ib_j|\Upsilon_{(+)\acute d\mu} >\Big] \\
&= \frac{4}{3}\theta^{(120)(-)}_{\acute a\acute b,\acute c\acute d}\Big[4\mathbf{P}^{ij\mathbf{T}}_{\acute a\nu}\mathbf{P}_{\acute b\nu}\mathbf{Q}^{\mathbf{T}}_{\acute cij\mu}\mathbf{Q}_{\acute d\mu} - 4\mathbf{P}^{\mathbf{T}}_{\acute ai\nu}\mathbf{P}_{\acute bj\nu}\mathbf{Q}^{i\mathbf{T}}_{\acute c\mu}\mathbf{Q}^{j}_{\acute d\mu} \\
&\qquad -4\mathbf{P}^{ij\mathbf{T}}_{\acute a\nu}\mathbf{P}^{kl}_{\acute b\nu}\mathbf{Q}^{\mathbf{T}}_{\acute cij\mu}\mathbf{Q}_{\acute dkl\mu} - 8\mathbf{P}^{ij\mathbf{T}}_{\acute a\nu}\mathbf{P}^{kl}_{\acute b\nu}\mathbf{Q}^{\mathbf{T}}_{\acute cjk\mu}\mathbf{Q}_{\acute dil\mu} \\
&\qquad +\epsilon_{ijklm}\mathbf{P}^{ij\mathbf{T}}_{\acute a\nu}\mathbf{P}^{kl}_{\acute b\nu}\mathbf{Q}^{\mathbf{T}}_{\acute c\mu}\mathbf{Q}^{m}_{\acute d\mu} - 4\epsilon^{ijklm}\mathbf{P}^{\mathbf{T}}_{\acute a\nu}\mathbf{P}_{\acute bi\nu}\mathbf{Q}^{\mathbf{T}}_{\acute cjk\mu}\mathbf{Q}_{\acute dlm\mu} \\
&\qquad +8\mathbf{P}^{\mathbf{T}}_{\acute a\nu}\mathbf{P}_{\acute bi\nu}\mathbf{Q}^{\mathbf{T}}_{\acute c\mu}\mathbf{Q}^{i}_{\acute d\mu} + 4\mathbf{P}^{\mathbf{T}}_{\acute ai\nu}\mathbf{P}^{jk}_{\acute b\nu}\mathbf{Q}^{i\mathbf{T}}_{\acute c\mu}\mathbf{Q}_{\acute djk\mu} \\
&\qquad -4\mathbf{P}^{\mathbf{T}}_{\acute ai\nu}\mathbf{P}^{ij}_{\acute b\nu}\mathbf{Q}^{k\mathbf{T}}_{\acute c\mu}\mathbf{Q}_{\acute dkj\mu}\Big]
\end{aligned}
\tag{C.39}
$$

where

$$
\theta^{(120)(-)}_{\acute a\acute b,\acute c\acute d} = h^{(120)(-)}_{\acute a\acute b}\,\bar h^{(120)(-)}_{\acute c\acute d}\,k^{(120)}_{\mathcal V}\,\widetilde{\mathcal M}^{(120)}_{\mathcal V\mathcal V'}\bar k^{(120)}_{\mathcal U'}
\tag{C.40}
$$

The $SU(5)\times U(1)$ decomposition of the quartic couplings can be carried out using the results given in appendix F.

C.10 The $\left(\overline{\mathbf{144}}\times\overline{\mathbf{144}}\right)_{\overline{\mathbf{126}}}(\mathbf{144}\times\mathbf{144})_{\mathbf{126}}$ couplings

Here we begin by considering the superpotential

$$
\begin{aligned}
\mathsf{W}^{(126,\overline{126})'} &= \frac{1}{2}\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal W}\mathcal M^{(126,\overline{126})}_{\mathcal W\mathcal W'}\overline\Phi_{\rho\nu\sigma\lambda\vartheta\mathcal W'} \\
&+\frac{1}{5!}h^{(\overline{126})}_{\acute a\acute b} < \Upsilon^{*}_{(+)\acute a\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Upsilon_{(+)\acute b\mu} > k^{(\overline{126})}_{\mathcal W}\overline\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal W}
\end{aligned}
$$

$$+\frac{1}{5!}\bar{h}^{(126)}_{\acute{a}\acute{b}} < \Upsilon^{*}_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(126)}_{\mathcal{W}}\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}$$

$$+\frac{1}{5!}\bar{h}^{(126)}_{\acute{a}\acute{b}} < \Upsilon^{*}_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(126)}_{\mathcal{W}}\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}} \tag{C.41}$$

Eliminating $\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}$, $\overline{\Phi}_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}$ through the F flatness conditions

$$\frac{\partial \mathsf{W}^{(126,\overline{126})'}}{\partial\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}}=0, \qquad \frac{\partial \mathsf{W}^{(126,\overline{126})'}}{\partial\overline{\Phi}_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}}=0 \tag{C.42}$$

gives the quartic interaction below

$$\begin{aligned}
\mathsf{W}^{(\overline{144}\times\overline{144})_{\overline{126}}(144\times 144)_{126}} &= \frac{1}{7200}\kappa^{(126,\overline{126})}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Upsilon^{*}_{(+)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Upsilon_{(+)\acute{b}\mu} > \\
&\qquad \times < \Upsilon^{*}_{(-)\acute{c}\tau}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Upsilon_{(-)\acute{d}\tau} > \\
&= \frac{2}{15}\kappa^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[2\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}_{\acute{d}\nu} - 2\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{ij}_{\acute{b}\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}_{\acute{d}ij\nu} \\
&\quad +2\mathbf{P}^{\mathbf{T}}_{\acute{a}i\mu}\mathbf{P}_{\acute{b}j\mu}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}^{j}_{\acute{d}\nu} + 48\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}k\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}^{k}_{\acute{d}\nu} \\
&\quad -2\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}k\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\nu}\mathbf{Q}^{k}_{\acute{d}\nu} + \mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}j\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\nu}\mathbf{Q}^{k}_{\acute{d}\nu} \\
&\quad +6\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\nu}\mathbf{Q}_{\acute{d}kl\nu} - 30\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\nu}\mathbf{Q}_{\acute{d}jl\nu} \\
&\quad +9\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}il\nu}\mathbf{Q}_{\acute{d}jk\nu} + 2\epsilon^{ijklm}\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}i\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\nu}\mathbf{Q}_{\acute{d}lm\nu} \\
&\quad +2\epsilon_{ijklm}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}^{m}_{\acute{d}\nu}\Big]
\end{aligned} \tag{C.43}$$

where

$$\kappa^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} = h^{(\overline{126})(+)}_{\acute{a}\acute{b}}\bar{h}^{(126)(+)}_{\acute{c}\acute{d}}\bar{k}^{(126)}_{\mathcal{W}}\widehat{\mathcal{M}}^{(126,\overline{126})}_{\mathcal{W}\mathcal{W}'}k^{(\overline{126})}_{\mathcal{W}'} \tag{C.44}$$

and

$$\widehat{\mathcal{M}}^{(126,\overline{126})} = \left(\mathcal{M}^{(126,\overline{126})}\right)^{-1}\left[\left(\mathcal{M}^{(126,\overline{126})}\right)^{\mathbf{T}}\left(\mathcal{M}^{(126,\overline{126})}\right)^{-1} - 2\cdot\mathbf{1}\right] \tag{C.45}$$

C.11 The $\left(\overline{\mathbf{144}}\times\overline{\mathbf{144}}\right)_{\overline{\mathbf{126}}}\left(\overline{\mathbf{16}}\times\overline{\mathbf{16}}\right)_{\mathbf{126}}$ couplings

The analysis here follows a very similar approach as above and one gets

$$\begin{aligned}
\mathsf{W}^{(\overline{144}\times\overline{144})_{\overline{126}}(\overline{16}\times\overline{16})_{126}} &= \frac{1}{7200}\varsigma^{(126,\overline{126})}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Upsilon^{*}_{(+)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Upsilon_{(+)\acute{b}\mu} > \\
&\qquad \times < \Psi^{*}_{(-)\acute{c}}|B\Gamma_{[\nu}\Gamma_{\rho}\Gamma_{\sigma}\Gamma_{\lambda}\Gamma_{\vartheta]}|\Psi_{(-)\acute{d}} > \\
&= \frac{2}{15}\varsigma^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[2\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}}\mathbf{N}_{\acute{d}} - 2\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{ij}_{\acute{b}\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}}\mathbf{N}_{\acute{d}ij} \\
&\quad +2\mathbf{P}^{\mathbf{T}}_{\acute{a}i\mu}\mathbf{P}_{\acute{b}j\mu}\mathbf{N}^{i\mathbf{T}}_{\acute{c}}\mathbf{N}^{j}_{\acute{d}} + 48\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}k\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}}\mathbf{N}^{k}_{\acute{d}} \\
&\quad -2\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}k\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}ij}\mathbf{N}^{k}_{\acute{d}} + \mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}j\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}ik}\mathbf{N}^{k}_{\acute{d}} \\
&\quad +6\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}ij}\mathbf{N}_{\acute{d}kl} - 30\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}ik}\mathbf{N}_{\acute{d}jl} \\
&\quad +9\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}il}\mathbf{N}_{\acute{d}jk} + 2\epsilon^{ijklm}\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}i\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}jk}\mathbf{N}_{\acute{d}lm} \\
&\quad +2\epsilon_{ijklm}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu}\mathbf{N}^{\mathbf{T}}_{\acute{c}}\mathbf{N}^{m}_{\acute{d}}\Big]
\end{aligned} \tag{C.46}$$

where

$$\varsigma^{(126,\overline{126})(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} = h^{(\overline{126})(+)}_{\acute{a}\acute{b}} \bar{f}^{(126)(+)}_{\acute{c}\acute{d}} \bar{l}^{(126)}_{\mathcal{W}} \widehat{\mathcal{M}}^{(126,\overline{126})}_{\mathcal{W}\mathcal{W}'} k^{(\overline{126})}_{\mathcal{W}'} \tag{C.47}$$

Further decomposition in the $SU(5) \times U(1)$ basis can be carried out using the results in appendix G.

D. Matter-Higgs couplings

In this appendix we evaluate the quartic couplings involving two semi-spinors of matter fields, i.e., the 16 plets of matter fields and two vector-spinor fields. We will utilize the analysis of section 4 to compute these quartic couplings. Thus the couplings would arise by mediation from 10, 120 and $126 + \overline{126}$ between the matter sector and the Higgs sector. We discuss now these computations in detail below.

D.1 The $(\mathbf{16} \times \mathbf{16})_{\mathbf{10}}\ \left(\overline{\mathbf{144}} \times \overline{\mathbf{144}}\right)_{\mathbf{10}}$ couplings

For the computation of the $(16 \times 16)_{10}\ \left(\overline{144} \times \overline{144}\right)_{10}$ couplings arising from the 10 mediation we consider the superpotential

$$\begin{aligned} \mathsf{W}^{(10)''} &= \frac{1}{2}\Phi_{\nu\mathcal{U}}\mathcal{M}^{(10)}_{\mathcal{U}\mathcal{U}'}\Phi_{\nu\mathcal{U}'} + h^{(10)}_{\acute{a}\acute{b}} < \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_\nu|\Upsilon_{(+)\acute{b}\mu} > k^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}} \\ &+ f^{(10)}_{\acute{a}\acute{b}} < \Psi^*_{(+)\acute{a}}|B\Gamma_\nu|\Psi_{(+)\acute{b}} > l^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}} + \bar{h}^{(10)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_\nu|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(10)}_{\mathcal{U}}\Phi_{\nu\mathcal{U}} \end{aligned} \tag{D.1}$$

Eliminating $\Phi_{\nu\mathcal{U}}$ using F flatness condition we get

$$\begin{aligned} \mathsf{W}^{(16\times 16)_{10}(\overline{144}\times\overline{144})_{10}} &= -2\xi^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Psi^*_{(+)\acute{a}}|B\Gamma_\rho|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|\Gamma_\rho|\Upsilon_{(+)\acute{d}\nu} > \\ &= -4\xi^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big[< \Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(+)\acute{d}\nu} > \\ &\qquad + < \Psi^*_{(+)\acute{a}}|Bb_i^\dagger|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb_i|\Upsilon_{(+)\acute{d}\nu} >\Big] \\ &= 2\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big(\epsilon_{jklmn}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{P}^{kl\mathbf{T}}_{\acute{c}\mu}\mathbf{P}^{mn}_{\acute{d}\mu} - 8\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}j\mu}\mathbf{P}_{\acute{d}\mu} \\ &\qquad + \epsilon_{jklmn}\mathbf{M}^{kl\mathbf{T}}_{\acute{a}}\mathbf{M}^{mn}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}i\mu}\mathbf{P}^{ij}_{\acute{d}\mu} - 8\mathbf{M}^{\mathbf{T}}_{\acute{a}j}\mathbf{M}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}i\mu}\mathbf{P}^{ij}_{\acute{d}\mu}\Big) \end{aligned} \tag{D.2}$$

where

$$\xi^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} = f^{(10)(+)}_{\acute{a}\acute{b}} h^{(10)(+)}_{\acute{c}\acute{d}} l^{(10)}_{\mathcal{U}} \widetilde{\mathcal{M}}^{(10)}_{\mathcal{U}\mathcal{U}'} k^{(10)}_{\mathcal{U}'} \tag{D.3}$$

D.2 The $(\mathbf{16} \times \mathbf{16})_{\mathbf{10}}\ (\mathbf{144} \times \mathbf{144})_{\mathbf{10}}$ couplings

An analysis similar to the above gives

$$\begin{aligned} \mathsf{W}^{(16\times 16)_{10}(144\times 144)_{10}} &= -2\zeta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Psi^*_{(+)\acute{a}}|B\Gamma_\rho|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_\rho|\Upsilon_{(-)\acute{d}\nu} > \\ &= -4\zeta^{(10)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big[< \Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute{d}\nu} > \\ &\qquad + < \Psi^*_{(+)\acute{a}}|Bb_i^\dagger|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(-)\acute{c}\nu}|Bb_i|\Upsilon_{(-)\acute{d}\nu} >\Big] \end{aligned}$$

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$$
\begin{aligned}
&= 2\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \Big(8\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{Q}^{k\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}kj\mu} - 8\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu} \\
&\quad -\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}kl\mu}\mathbf{Q}_{\acute{d}ij\mu} + \mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\mu}\mathbf{Q}_{\acute{d}jl\mu} \\
&\quad -\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}il\mu}\mathbf{Q}_{\acute{d}jk\mu} + \epsilon^{ijklm}\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\mu}\mathbf{Q}_{\acute{d}lm\mu} \\
&\quad +\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{m\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu}\Big) \qquad \text{(D.4)}
\end{aligned}
$$

where

$$
\zeta^{(10)(+)}_{\acute{a}\acute{b},\acute{c}\acute{d}} = f^{(10)(+)}_{\acute{a}\acute{b}}\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}\, l^{(10)}_{\mathcal{U}}\,\widetilde{\mathcal{M}}^{(10)}_{\mathcal{U}\mathcal{U}'}\bar{k}^{(10)}_{\mathcal{U}'} \qquad \text{(D.5)}
$$

To obtain the reduction of the results of sections D.1 and D.2 in the $SU(5)\times U(1)$ basis we use the results of appendix E.

D.3 The $(\mathbf{16}\times\mathbf{16})_{\mathbf{120}}\,(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{120}}$ couplings

Here we begin by considering the superpotential

$$
\begin{aligned}
\mathsf{W}^{(120)''} &= \frac{1}{2}\Phi_{\nu\rho\lambda V}\mathcal{M}^{(120)}_{VV'}\Phi_{\rho\nu\lambda V'} + \frac{1}{3!}h^{(120)}_{\acute{a}\acute{b}} < \Upsilon^*_{(+)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(+)\acute{b}\mu} > k^{(120)}_V\Phi_{\nu\rho\lambda V} \\
&\quad + \frac{1}{3!}f^{(120)}_{\acute{a}\acute{b}} < \Psi^*_{(+)\acute{a}}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Psi_{(+)\acute{b}} > l^{(120)}_V\Phi_{\nu\rho\lambda V} \\
&\quad + \frac{1}{3!}\bar{h}^{(120)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(120)}_V\Phi_{\nu\rho\lambda V} \qquad \text{(D.6)}
\end{aligned}
$$

Eliminating $\Phi_{\nu\rho\lambda V}$ by the F flatness condition we get

$$
\begin{aligned}
\mathsf{W}^{(16\times16)_{120}(\overline{144}\times\overline{144})_{120}} &= -\frac{1}{18}\xi^{(120)}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Psi^*_{(+)\acute{a}}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(+)\acute{d}\nu} > \\
&= -\frac{1}{18}\xi^{(120)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[< \Psi^*_{(+)\acute{a}}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad -28 < \Psi^*_{(+)\acute{a}}|B\Gamma_\nu|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|B\Gamma_\nu|\Upsilon_{(+)\acute{d}\nu} >\Big] \\
&= -\frac{4}{9}\xi^{(120)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[< \Psi^*_{(+)\acute{a}}|Bb_ib_jb_k|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb^\dagger_ib^\dagger_jb^\dagger_k|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad -6 < \Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb^\dagger_i|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad +3 < \Psi^*_{(+)\acute{a}}|Bb^\dagger_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb^\dagger_nb_nb_i|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad +3 < \Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb^\dagger_ib^\dagger_nb_n|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad +3 < \Psi^*_{(+)\acute{a}}|Bb^\dagger_ib^\dagger_jb_k|\Psi_{(+)\acute{b}} >< \Upsilon^*_{(+)\acute{c}\nu}|Bb^\dagger_kb_ib_j|\Upsilon_{(+)\acute{d}\nu} > \\
&\quad + < \Upsilon^*_{(+)\acute{a}\nu}|Bb_ib_jb_k|\Upsilon_{(+)\acute{b}\nu} >< \Psi^*_{(+)\acute{c}}|Bb^\dagger_ib^\dagger_jb^\dagger_k|\Psi_{(+)\acute{d}} > \\
&\quad -6 < \Upsilon^*_{(+)\acute{a}\nu}|Bb_i|\Upsilon_{(+)\acute{b}\nu} >< \Psi^*_{(+)\acute{c}}|Bb^\dagger_i|\Psi_{(+)\acute{d}} > \\
&\quad +3 < \Upsilon^*_{(+)\acute{a}\nu}|Bb^\dagger_i|\Upsilon_{(+)\acute{b}\nu} >< \Psi^*_{(+)\acute{c}}|Bb^\dagger_nb_nb_i|\Psi_{(+)\acute{d}} > \\
&\quad +3 < \Upsilon^*_{(+)\acute{a}\nu}|Bb_i|\Upsilon_{(+)\acute{b}\nu} >< \Psi^*_{(+)\acute{c}}|Bb^\dagger_ib^\dagger_nb_n|\Psi_{(+)\acute{d}} > \\
&\quad +3 < \Upsilon^*_{(+)\acute{a}\nu}|Bb^\dagger_ib^\dagger_jb_k|\Upsilon_{(+)\acute{b}\nu} >< \Psi^*_{(+)\acute{c}}|Bb^\dagger_kb_ib_j|\Psi_{(+)\acute{d}} >\Big] \\
&= \frac{4}{3}\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[4\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}j}\mathbf{P}^{ij\mathbf{T}}_{\acute{c}\nu}\mathbf{P}_{\acute{d}\nu} + 4\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}i\nu}\mathbf{P}_{\acute{d}j\nu}
\end{aligned}
$$

$$
\begin{aligned}
&-4\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathbf{P}^{ij\mathbf{T}}_{\acute{c}\nu}\mathbf{P}_{\acute{d}j\nu}-4\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}j}\mathbf{P}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{P}_{\acute{d}i\nu}\\
&-\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kn}_{\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{c}n\nu}\mathbf{P}^{lm}_{\acute{d}\nu}-\epsilon_{ijklm}\mathbf{M}^{\mathbf{T}}_{\acute{a}n}\mathbf{M}^{lm}_{\acute{b}}\mathbf{P}^{ij\mathbf{T}}_{\acute{c}\nu}\mathbf{P}^{kn}_{\acute{d}\nu}\Big]
\end{aligned}
\tag{D.7}
$$

where

$$
\xi^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}=f^{(120)(-)}_{\acute{a}\acute{b}}h^{(120)(-)}_{\acute{c}\acute{d}}l^{(120)}_{V}\widetilde{\mathcal{M}}^{(120)}_{VV'}k^{(120)}_{V'}
\tag{D.8}
$$

D.4 The $(\mathbf{16}\times\mathbf{16})_{\mathbf{120}}(\mathbf{144}\times\mathbf{144})_{\mathbf{120}}$ couplings

An analysis similar to the above gives

$$
\begin{aligned}
\mathsf{W}^{(16\times16)_{120}(144\times144)_{120}}=&-\frac{1}{18}\zeta^{(120)}_{\acute{a}\acute{b},\acute{c}\acute{d}}<\Psi^*_{(+)\acute{a}}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_{\lambda]}|\Upsilon_{(-)\acute{d}\nu}>\\
=&-\frac{1}{18}\zeta^{(120)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[<\Psi^*_{(+)\acute{a}}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_\nu\Gamma_\rho\Gamma_\lambda|\Upsilon_{(-)\acute{d}\nu}>\\
&-28<\Psi^*_{(+)\acute{a}}|B\Gamma_\nu|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|B\Gamma_\nu|\Upsilon_{(-)\acute{d}\nu}>\Big]\\
=&-\frac{4}{9}\zeta^{(120)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[<\Psi^*_{(+)\acute{a}}|Bb_ib_jb_k|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger b_j^\dagger b_k^\dagger|\Upsilon_{(-)\acute{d}\nu}>\\
&-6<\Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute{d}\nu}>\\
&+3<\Psi^*_{(+)\acute{a}}|Bb_i^\dagger|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|Bb_n^\dagger b_nb_i|\Upsilon_{(-)\acute{d}\nu}>\\
&+3<\Psi^*_{(+)\acute{a}}|Bb_i|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|Bb_i^\dagger b_n^\dagger b_n|\Upsilon_{(-)\acute{d}\nu}>\\
&+3<\Psi^*_{(+)\acute{a}}|Bb_i^\dagger b_j^\dagger b_k|\Psi_{(+)\acute{b}}><\Upsilon^*_{(-)\acute{c}\nu}|Bb_k^\dagger b_ib_j|\Upsilon_{(-)\acute{d}\nu}>\\
&+<\Upsilon^*_{(-)\acute{a}\nu}|Bb_ib_jb_k|\Upsilon_{(-)\acute{b}\nu}><\Psi^*_{(+)\acute{c}}|Bb_i^\dagger b_j^\dagger b_k^\dagger|\Psi_{(+)\acute{d}}>\\
&-6<\Upsilon^*_{(-)\acute{a}\nu}|Bb_i|\Upsilon_{(-)\acute{b}\nu}><\Psi^*_{(+)\acute{c}}|Bb_i^\dagger|\Psi_{(+)\acute{d}}>\\
&+3<\Upsilon^*_{(-)\acute{a}\nu}|Bb_i^\dagger|\Upsilon_{(-)\acute{b}\nu}><\Psi^*_{(+)\acute{c}}|Bb_n^\dagger b_nb_i|\Psi_{(+)\acute{d}}>\\
&+3<\Upsilon^*_{(-)\acute{a}\nu}|Bb_i|\Upsilon_{(-)\acute{b}\nu}><\Psi^*_{(+)\acute{c}}|Bb_i^\dagger b_n^\dagger b_n|\Psi_{(+)\acute{d}}>\\
&+3<\Upsilon^*_{(-)\acute{a}\nu}|Bb_i^\dagger b_j^\dagger b_k|\Upsilon_{(-)\acute{b}\nu}><\Psi^*_{(+)\acute{c}}|Bb_k^\dagger b_ib_j|\Psi_{(+)\acute{d}}>\Big]\\
=&\frac{4}{3}\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[4\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\mu}\mathbf{Q}_{\acute{d}\mu}-4\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}j}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}^{j}_{\acute{d}\mu}\\
&-4\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\mu}\mathbf{Q}_{\acute{d}kl\mu}-8\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\mu}\mathbf{Q}_{\acute{d}il\mu}\\
&+\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}^{m}_{\acute{d}\mu}-4\epsilon^{ijklm}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\mu}\mathbf{Q}_{\acute{d}lm\mu}\\
&+8\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}^{i}_{\acute{d}\mu}+4\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{jk}_{\acute{b}}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}jk\mu}\\
&-4\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}^{ij}_{\acute{b}}\mathbf{Q}^{k\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}kj\mu}\Big]
\end{aligned}
\tag{D.9}
$$

where

$$
\zeta^{(120)(-)}_{\acute{a}\acute{b},\acute{c}\acute{d}}=f^{(120)(-)}_{\acute{a}\acute{b}}\bar{h}^{(120)(-)}_{\acute{c}\acute{d}}l^{(120)}_{\mathcal{U}}\widetilde{\mathcal{M}}^{(120)}_{VV'}\bar{k}^{(120)}_{V'}
\tag{D.10}
$$

Further reduction of the above to the $SU(5)\times U(1)$ basis can be achieved by using appendix F.

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D.5 The $(16 \times 16)_{\overline{126}}$ $(144 \times 144)_{126}$ couplings

Finally we consider the matter-Higgs couplings via $126 + \overline{126}$ mediation. Here we begin by considering the superpotential

$$\begin{aligned}
\mathsf{W}^{(126,\overline{126})''} &= \frac{1}{2}\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}\mathcal{M}^{(126,\overline{126})}_{\mathcal{W}\mathcal{W}'}\overline{\Phi}_{\rho\nu\sigma\lambda\vartheta\mathcal{W}'} \\
&+\frac{1}{5!}f^{(\overline{126})}_{\acute{a}\acute{b}} < \Psi^*_{(+)\acute{a}}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_\sigma\Gamma_\lambda\Gamma_{\vartheta]}|\Psi_{(+)\acute{b}} > l^{(\overline{126})}_{\mathcal{W}}\overline{\Phi}_{\nu\rho\sigma\lambda\vartheta\mathcal{W}} \\
&+\frac{1}{5!}\bar{h}^{(126)}_{\acute{a}\acute{b}} < \Upsilon^*_{(-)\acute{a}\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_\sigma\Gamma_\lambda\Gamma_{\vartheta]}|\Upsilon_{(-)\acute{b}\mu} > \bar{k}^{(126)}_{\mathcal{W}}\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}
\end{aligned} \tag{D.11}$$

Eliminating $\Phi_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}$, $\overline{\Phi}_{\nu\rho\sigma\lambda\vartheta\mathcal{W}}$, by use of F flatness gives

$$\begin{aligned}
\mathsf{W}^{(16\times16)_{\overline{126}}(144\times144)_{126}} &= \frac{1}{7200}\varrho^{(126,\overline{126})}_{\acute{a}\acute{b},\acute{c}\acute{d}} < \Psi^*_{(+)\acute{a}}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_\sigma\Gamma_\lambda\Gamma_{\vartheta]}|\Psi_{(+)\acute{b}} > \\
&\quad \times < \Upsilon^*_{(-)\acute{c}\mu}|B\Gamma_{[\nu}\Gamma_\rho\Gamma_\sigma\Gamma_\lambda\Gamma_{\vartheta]}|\Upsilon_{(-)\acute{d}\mu} > \\
&= \frac{2}{15}\varrho^{(126,\overline{126})}_{\acute{a}\acute{b},\acute{c}\acute{d}}\Big[2\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}\mu} - 2\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}^{ij}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}ij\mu} \\
&\quad +2\mathbf{M}^{\mathbf{T}}_{\acute{a}i}\mathbf{M}_{\acute{b}j}\mathbf{Q}^{i\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}^{j}_{\acute{d}\mu} + 48\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}k}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}^{k}_{\acute{d}\mu} \\
&\quad -2\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}k}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\mu}\mathbf{Q}^{k}_{\acute{d}\mu} + \mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}j}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\mu}\mathbf{Q}^{k}_{\acute{d}\mu} \\
&\quad +6\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\mu}\mathbf{Q}_{\acute{d}kl\mu} - 30\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ik\mu}\mathbf{Q}_{\acute{d}jl\mu} \\
&\quad +9\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}il\mu}\mathbf{Q}_{\acute{d}jk\mu} + 2\epsilon^{ijklm}\mathbf{M}^{\mathbf{T}}_{\acute{a}}\mathbf{M}_{\acute{b}i}\mathbf{Q}^{\mathbf{T}}_{\acute{c}jk\mu}\mathbf{Q}_{\acute{d}lm\mu} \\
&\quad +2\epsilon_{ijklm}\mathbf{M}^{ij\mathbf{T}}_{\acute{a}}\mathbf{M}^{kl}_{\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}^{m}_{\acute{d}\mu}\Big]
\end{aligned} \tag{D.12}$$

where

$$\varrho^{(126,\overline{126})}_{\acute{a}\acute{b},\acute{c}\acute{d}} = f^{(\overline{126})}_{\acute{a}\acute{b}}\bar{h}^{(126)}_{\acute{c}\acute{d}}\bar{k}^{(126)}_{\mathcal{W}}\widehat{\mathcal{M}}^{(126,\overline{126})}_{\mathcal{W}\mathcal{W}'}l^{(\overline{126})}_{\mathcal{W}'} \tag{D.13}$$

A further reduction of the quartic interactions to the $SU(5)\times U(1)$ basis can be achieved by use of appendix G.

The quartic couplings discussed in appendices C, D are obtained by integrating out the intermediate fields which belong to the set of tensor representations 1, 45, 210, 10, 120, $126+\overline{126}$. The analysis given in the paper is quite general allowing for an arbitrary number of such intermediate tensor set. The results, however, can be simplified if one assumes just a single tensor field for each term in the set listed above. In this case the couplings show a factorization. This case can be gotten from the analysis of the paper by the following simple algorithm of replacemen $\bar{\lambda}^{(\cdot)}_{\acute{a}\acute{b},\acute{c}\acute{d}} \quad \rightarrow \quad -\frac{1}{4}\frac{\bar{h}^{(\cdot)}_{\acute{a}\acute{b}}\bar{h}^{(\cdot)}_{\acute{c}\acute{d}}}{\mathcal{M}}$ and similarly for other couplings.

E. Details of couplings from 10-plet mediation

In this appendix we expand the $SO(10)$ coupling structures that enter in 10-plet mediation in section 4 and appendicies C and D in a $SU(5) \times U(1)$ basis. We list these structures below

$$h^{(10)(+)}_{\acute{c}\acute{d}}\mathbf{P}^{\mathbf{T}}_{\acute{c}j\nu}\mathbf{P}_{\acute{d}\nu}=h^{(10)(+)}_{\acute{c}\acute{d}}\left[\frac{\sqrt{6}}{5}\mathcal{P}^{\mathbf{T}}_{\acute{c}jk}\mathcal{P}^{k}_{\acute{d}}+\sqrt{\frac{2}{5}}\mathcal{P}^{(S)\mathbf{T}}_{\acute{c}jk}\mathcal{P}^{k}_{\acute{d}}+\mathcal{P}^{k\mathbf{T}}_{\acute{c}j}\mathcal{P}_{\acute{d}k}\right] \tag{E.1}$$

$$h^{(10)(+)}_{\acute{c}\acute{d}}\epsilon_{jklmn}\mathbf{P}^{kl\mathbf{T}}_{\acute{c}\nu}\mathbf{P}^{mn}_{\acute{d}\nu}=h^{(10)(+)}_{\acute{c}\acute{d}}\left[4\mathcal{P}^{pq\mathbf{T}}_{\acute{c}r}\mathcal{P}^{r}_{\acute{d}pqj}-4\sqrt{\frac{2}{15}}\mathcal{P}^{\mathbf{T}}_{\acute{c}pq}\mathcal{P}^{pq}_{\acute{d}j}-\frac{4\sqrt{6}}{5}\mathcal{P}^{p\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}pj}\right] \tag{E.2}$$

$$\begin{aligned}h^{(10)(+)}_{\acute{a}\acute{b}}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\nu}\mathbf{P}^{kl}_{\acute{b}\nu}=h^{(10)(+)}_{\acute{a}\acute{b}}&\left[\frac{1}{6}\epsilon^{ijpqr}\mathcal{P}^{s\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{kl}_{\acute{b}s}+\frac{1}{6}\epsilon^{klpqr}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}s}\mathcal{P}^{s}_{\acute{b}pqr}\right.\\&+\frac{1}{12\sqrt{5}}\epsilon^{ijpqr}\mathcal{P}^{k\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{l}_{\acute{b}}-\frac{1}{12\sqrt{5}}\epsilon^{klpqr}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}pqr}\\&-\frac{1}{12\sqrt{5}}\epsilon^{ijpqr}\mathcal{P}^{l\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{k}_{\acute{b}}+\frac{1}{12\sqrt{5}}\epsilon^{klpqr}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{i}_{\acute{b}pqr}\\&+\frac{1}{10\sqrt{6}}\epsilon^{ijlpq}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{k}_{\acute{b}}-\frac{1}{10\sqrt{6}}\epsilon^{klipq}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}pq}\\&-\frac{1}{10\sqrt{6}}\epsilon^{ijkpq}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{l}_{\acute{b}}+\frac{1}{10\sqrt{6}}\epsilon^{kljpq}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}pq}\\&\left.-\frac{1}{\sqrt{30}}\epsilon^{ijpqr}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{kl}_{\acute{b}r}-\frac{1}{\sqrt{30}}\epsilon^{klpqr}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}p}\mathcal{P}_{\acute{b}qr}\right]\end{aligned} \tag{E.3}$$

$$\begin{aligned}\bar{h}^{(10)(+)}_{\acute{a}\acute{b}}\mathbf{Q}^{i\mathbf{T}}_{\acute{a}\mu}\mathbf{Q}_{\acute{b}ij\mu}=\bar{h}^{(10)(+)}_{\acute{a}\acute{b}}&\left[-\frac{1}{2\sqrt{30}}\epsilon_{jklmn}\mathcal{Q}^{kp\mathbf{T}}_{\acute{a}}\mathcal{Q}^{lmn}_{\acute{b}p}-\frac{1}{6\sqrt{2}}\epsilon_{jklmn}\mathcal{Q}^{kp\mathbf{T}}_{(S)\acute{a}}\mathcal{Q}^{lmn}_{\acute{b}p}\right.\\&\left.+\frac{1}{10}\epsilon_{jklmn}\mathcal{Q}^{kl\mathbf{T}}_{\acute{a}}\mathcal{Q}^{mn}_{\acute{b}}+\mathcal{Q}^{l\mathbf{T}}_{\acute{a}k}\mathcal{Q}^{k}_{\acute{b}lj}-\frac{1}{2\sqrt{5}}\mathcal{Q}^{k\mathbf{T}}_{\acute{a}j}\mathcal{Q}_{\acute{b}k}\right]\end{aligned} \tag{E.4}$$

$$\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}\mathbf{Q}^{j\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}_{\acute{d}\nu}=\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}\left[\frac{\sqrt{6}}{5}\mathcal{Q}^{jk\mathbf{T}}_{\acute{c}}\mathcal{Q}_{\acute{d}k}+\sqrt{\frac{2}{5}}\mathcal{Q}^{jk\mathbf{T}}_{(S)\acute{c}}\mathcal{Q}_{\acute{d}k}+\mathcal{Q}^{j\mathbf{T}}_{\acute{c}k}\mathcal{Q}^{k}_{\acute{d}}\right] \tag{E.5}$$

$$\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}\epsilon^{jklmn}\mathbf{Q}^{\mathbf{T}}_{\acute{c}kl\nu}\mathbf{Q}_{\acute{d}mn\nu}=\bar{h}^{(10)(+)}_{\acute{c}\acute{d}}\left[4\mathcal{Q}^{r\mathbf{T}}_{\acute{c}pq}\mathcal{Q}^{pqj}_{\acute{d}r}-4\sqrt{\frac{2}{15}}\mathcal{Q}^{pq\mathbf{T}}_{\acute{c}}\mathcal{Q}^{j}_{\acute{d}pq}-\frac{4\sqrt{6}}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{c}p}\mathcal{Q}^{pj}_{\acute{d}}\right] \tag{E.6}$$

$$\begin{aligned}\bar{h}^{(10)(+)}_{\acute{a}\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\nu}\mathbf{Q}_{\acute{d}kl\nu}=\bar{h}^{(10)(+)}_{\acute{a}\acute{b}}&\left[\frac{1}{6}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute{a}s}\mathcal{Q}^{s}_{\acute{b}kl}+\frac{1}{6}\epsilon_{klpqr}\mathcal{Q}^{s\mathbf{T}}_{\acute{a}ij}\mathcal{Q}^{pqr}_{\acute{b}s}\right.\\&+\frac{1}{12\sqrt{5}}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute{a}k}\mathcal{Q}_{\acute{b}l}-\frac{1}{12\sqrt{5}}\epsilon_{klpqr}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{Q}^{pqr}_{\acute{b}j}\\&-\frac{1}{12\sqrt{5}}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute{a}l}\mathcal{Q}_{\acute{b}k}+\frac{1}{12\sqrt{5}}\epsilon_{klpqr}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{Q}^{pqr}_{\acute{b}i}\\&+\frac{1}{10\sqrt{6}}\epsilon_{ijlpq}\mathcal{Q}^{pq\mathbf{T}}_{\acute{a}}\mathcal{Q}_{\acute{b}k}-\frac{1}{10\sqrt{6}}\epsilon_{klipq}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{Q}^{pq}_{\acute{b}}\\&-\frac{1}{10\sqrt{6}}\epsilon_{ijkpq}\mathcal{Q}^{pq\mathbf{T}}_{\acute{a}}\mathcal{Q}_{\acute{b}l}+\frac{1}{10\sqrt{6}}\epsilon_{kljpq}\mathcal{Q}^{\mathbf{T}}_{\acute{a}i}\mathcal{Q}^{pq}_{\acute{b}}\\&\left.-\frac{1}{\sqrt{30}}\epsilon_{ijpqr}\mathcal{Q}^{pq\mathbf{T}}_{\acute{a}}\mathcal{Q}^{r}_{\acute{b}kl}-\frac{1}{\sqrt{30}}\epsilon_{klpqr}\mathcal{Q}^{p\mathbf{T}}_{\acute{a}ij}\mathcal{Q}^{qr}_{\acute{b}}\right]\end{aligned} \tag{E.7}$$

F. Details of couplings from 120-plet mediation

In this appendix we expand the $SO(10)$ coupling structures that enter in 120-plet mediation in section 4 and appendicies C and D in a $SU(5)\times U(1)$ basis. We list these structures below

$$\begin{aligned} h^{(120)(-)}_{\acute{c}\acute{d}}\mathbf{P}^{\mathbf{T}}_{\acute{c}n\mu}\mathbf{P}^{lm}_{\acute{d}\mu} = h^{(120)(-)}_{\acute{c}\acute{d}}\Bigg[&\frac{1}{2\sqrt{30}}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{c}np}\mathcal{P}^{p}_{\acute{d}ijk} - \frac{1}{10}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{c}ni}\mathcal{P}_{\acute{d}jk} \\ &+\frac{1}{6\sqrt{2}}\epsilon^{ijklm}\mathcal{P}^{(S)\mathbf{T}}_{\acute{c}np}\mathcal{P}^{p}_{\acute{d}ijk} - \frac{1}{2\sqrt{15}}\epsilon^{ijklm}\mathcal{P}^{(S)\mathbf{T}}_{\acute{c}ni}\mathcal{P}_{\acute{d}jk} \\ &+\mathcal{P}^{k\mathbf{T}}_{\acute{c}n}\mathcal{P}^{lm}_{\acute{d}k} + \frac{1}{2\sqrt{5}}\mathcal{P}^{l\mathbf{T}}_{\acute{c}n}\mathcal{P}^{m}_{\acute{d}} - \frac{1}{2\sqrt{5}}\mathcal{P}^{m\mathbf{T}}_{\acute{c}n}\mathcal{P}^{l}_{\acute{d}}\Bigg] \end{aligned} \tag{F.1}$$

$$\begin{aligned} h^{(120)(-)}_{\acute{a}\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{a}i\mu}\mathbf{P}_{\acute{b}j\mu} = h^{(120)(-)}_{\acute{a}\acute{b}}\Bigg[&\sqrt{\frac{3}{10}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}jk} - \sqrt{\frac{3}{10}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}j}\mathcal{P}_{\acute{b}ik} \\ &+\sqrt{2}\mathcal{P}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}^{(S)}_{\acute{b}jk} - \sqrt{2}\mathcal{P}^{k\mathbf{T}}_{\acute{a}j}\mathcal{P}^{(S)}_{\acute{b}ik}\Bigg] \end{aligned} \tag{F.2}$$

$$\begin{aligned} h^{(120)(-)}_{\acute{c}\acute{d}}\mathbf{P}^{ij\mathbf{T}}_{\acute{c}\mu}\mathbf{P}_{\acute{d}\mu} = h^{(120)(-)}_{\acute{c}\acute{d}}\Bigg[&\frac{2}{\sqrt{5}}\mathcal{P}^{ij\mathbf{T}}_{\acute{c}k}\mathcal{P}^{k}_{\acute{d}} + \frac{2}{5}\mathcal{P}^{j\mathbf{T}}_{\acute{c}}\mathcal{P}^{i}_{\acute{d}} \\ &+\frac{1}{6}\epsilon^{ijklm}\mathcal{P}^{n\mathbf{T}}_{\acute{c}klm}\mathcal{P}_{\acute{d}n} - \frac{1}{\sqrt{30}}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{c}kl}\mathcal{P}_{\acute{d}m}\Bigg] \end{aligned} \tag{F.3}$$

$$h^{(120)(-)}_{\acute{a}\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}i\mu} = h^{(120)(-)}_{\acute{a}\acute{b}}\left[\frac{\sqrt{6}}{5}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}ij} + \sqrt{\frac{2}{5}}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}ij} + \mathcal{P}^{\mathbf{T}}_{\acute{a}j}\mathcal{P}^{j}_{\acute{b}i}\right] \tag{F.4}$$

$$\begin{aligned} h^{(120)(-)}_{\acute{c}\acute{d}}\mathbf{P}^{ij\mathbf{T}}_{\acute{c}\mu}\mathbf{P}_{\acute{d}j\mu} = h^{(120)(-)}_{\acute{c}\acute{d}}\Bigg[&\mathcal{P}^{ik\mathbf{T}}_{\acute{c}l}\mathcal{P}^{l}_{\acute{d}k} + \frac{1}{2\sqrt{5}}\mathcal{P}^{j\mathbf{T}}_{\acute{c}}\mathcal{P}^{i}_{\acute{d}j} \\ &+\frac{1}{2\sqrt{30}}\epsilon^{iklmn}\mathcal{P}^{p\mathbf{T}}_{\acute{c}lmn}\mathcal{P}_{\acute{d}kp} + \frac{1}{6\sqrt{2}}\epsilon^{iklmn}\mathcal{P}^{p\mathbf{T}}_{\acute{c}lmn}\mathcal{P}^{(S)}_{\acute{d}kp} \\ &-\frac{1}{10}\epsilon^{iklmn}\mathcal{P}^{\mathbf{T}}_{\acute{c}mn}\mathcal{P}^{(S)}_{\acute{d}kl}\Bigg] \end{aligned} \tag{F.5}$$

$$\begin{aligned} \bar{h}^{(120)(-)}_{\acute{a}\acute{b}}\epsilon^{ijklm}\mathbf{Q}^{\mathbf{T}}_{\acute{a}ij\mu}\mathbf{Q}_{\acute{b}kn\mu} = \bar{h}^{(120)(-)}_{\acute{a}\acute{b}}\Bigg[&4\mathcal{Q}^{qlm\mathbf{T}}_{\acute{a}p}\mathcal{Q}^{p}_{\acute{b}qn} + \frac{2}{\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}p}\mathcal{Q}^{plm}_{\acute{b}n} + 4\sqrt{\frac{2}{15}}\mathcal{Q}^{pm\mathbf{T}}_{\acute{a}}\mathcal{Q}^{l}_{\acute{b}pn} \\ &-4\sqrt{\frac{2}{15}}\mathcal{Q}^{pl\mathbf{T}}_{\acute{a}}\mathcal{Q}^{m}_{\acute{b}pn} + \frac{4}{5}\sqrt{\frac{2}{3}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}n}\mathcal{Q}^{lm}_{\acute{b}} \\ &+\mathcal{Q}^{r\mathbf{T}}_{\acute{a}pq}\left(\delta^{m}_{n}\mathcal{Q}^{pql}_{\acute{b}r} - \delta^{l}_{n}\mathcal{Q}^{pqm}_{\acute{b}r}\right) \\ &+\sqrt{\frac{2}{15}}\left(\delta^{l}_{n}\mathcal{Q}^{m\mathbf{T}}_{\acute{a}pq} - \delta^{m}_{n}\mathcal{Q}^{l\mathbf{T}}_{\acute{a}pq}\right)\mathcal{Q}^{pq}_{\acute{b}} \\ &+\frac{1}{5}\sqrt{\frac{2}{3}}\left(\delta^{l}_{n}\mathcal{Q}^{pm\mathbf{T}}_{\acute{a}} - \delta^{m}_{n}\mathcal{Q}^{pl\mathbf{T}}_{\acute{a}}\right)\mathcal{Q}_{\acute{b}p}\Bigg] \end{aligned} \tag{F.6}$$

$$\bar{h}^{(120)(-)}_{\acute{c}\acute{d}}\mathbf{Q}^{n\mathbf{T}}_{\acute{c}\mu}\mathbf{Q}_{\acute{d}lm\mu} = \bar{h}^{(120)(-)}_{\acute{c}\acute{d}}\Bigg[\frac{1}{2\sqrt{30}}\epsilon_{ijklm}\mathcal{Q}^{np\mathbf{T}}_{\acute{c}}\mathcal{Q}^{ijk}_{\acute{d}p} - \frac{1}{10}\epsilon_{ijklm}\mathcal{Q}^{ni\mathbf{T}}_{\acute{c}}\mathcal{Q}^{jk}_{\acute{d}}$$
$$+\frac{1}{6\sqrt{2}}\epsilon_{ijklm}\mathcal{Q}^{np\mathbf{T}}_{(S)\acute{c}}\mathcal{Q}^{ijk}_{\acute{d}p} - \frac{1}{2\sqrt{15}}\epsilon_{ijklm}\mathcal{Q}^{ni\mathbf{T}}_{(S)\acute{c}}\mathcal{Q}^{jk}_{\acute{d}}$$
$$+\mathcal{Q}^{n\mathbf{T}}_{\acute{c}k}\mathcal{Q}^{k}_{\acute{d}lm} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}l}\mathcal{Q}_{\acute{d}m} - \frac{1}{2\sqrt{5}}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}m}\mathcal{Q}_{\acute{d}l}\Bigg] \tag{F.7}$$

$$\bar{h}^{(120)(-)}_{\acute{a}\acute{b}}\mathbf{Q}^{i\mathbf{T}}_{\acute{a}\mu}\mathbf{Q}^{j}_{\acute{b}\mu} = \bar{h}^{(120)(-)}_{\acute{a}\acute{b}}\Bigg[\sqrt{\frac{3}{10}}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}k}\mathcal{Q}^{jk}_{\acute{b}} - \sqrt{\frac{3}{10}}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}k}\mathcal{Q}^{ik}_{\acute{b}}$$
$$+\sqrt{2}\mathcal{Q}^{i\mathbf{T}}_{\acute{a}k}\mathcal{Q}^{jk}_{(S)\acute{b}} - \sqrt{2}\mathcal{Q}^{j\mathbf{T}}_{\acute{a}k}\mathcal{Q}^{ik}_{(S)\acute{b}}\Bigg] \tag{F.8}$$

$$\bar{h}^{(120)(-)}_{\acute{c}\acute{d}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\mu}\mathbf{Q}_{\acute{d}\mu} = \bar{h}^{(120)(-)}_{\acute{c}\acute{d}}\Bigg[\frac{2}{\sqrt{5}}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}ij}\mathcal{Q}_{\acute{d}k} + \frac{2}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{c}j}\mathcal{Q}_{\acute{d}i}$$
$$+\frac{1}{6}\epsilon_{ijklm}\mathcal{Q}^{klm\mathbf{T}}_{\acute{c}n}\mathcal{Q}^{n}_{\acute{d}} - \frac{1}{\sqrt{30}}\epsilon_{ijklm}\mathcal{Q}^{kl\mathbf{T}}_{\acute{c}}\mathcal{Q}^{m}_{\acute{d}}\Bigg] \tag{F.9}$$

$$\bar{h}^{(120)(-)}_{\acute{a}\acute{b}}\mathbf{Q}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{Q}^{i}_{\acute{b}\mu} = \bar{h}^{(120)(-)}_{\acute{a}\acute{b}}\Bigg[\frac{\sqrt{6}}{5}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{Q}^{ij}_{\acute{b}} + \sqrt{\frac{2}{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{a}j}\mathcal{Q}^{ij}_{(S)\acute{b}} + \mathcal{Q}^{j\mathbf{T}}_{\acute{a}}\mathcal{Q}^{i}_{\acute{b}j}\Bigg] \tag{F.10}$$

$$\bar{h}^{(120)(-)}_{\acute{c}\acute{d}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}ij\mu}\mathbf{Q}^{j}_{\acute{d}\mu} = \bar{h}^{(120)(-)}_{\acute{c}\acute{d}}\Bigg[\mathcal{Q}^{l\mathbf{T}}_{\acute{c}ik}\mathcal{Q}^{k}_{\acute{d}l} + \frac{1}{2\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{c}j}\mathcal{Q}^{j}_{\acute{d}i}$$
$$+\frac{1}{2\sqrt{30}}\epsilon_{iklmn}\mathcal{Q}^{lmn\mathbf{T}}_{\acute{c}p}\mathcal{Q}^{kp}_{\acute{d}} + \frac{1}{6\sqrt{2}}\epsilon_{iklmn}\mathcal{Q}^{lmn\mathbf{T}}_{\acute{c}p}\mathcal{Q}^{kp}_{(S)\acute{d}}$$
$$-\frac{1}{10}\epsilon_{iklmn}\mathcal{Q}^{mn\mathbf{T}}_{\acute{c}}\mathcal{Q}^{kl}_{(S)\acute{d}}\Bigg] \tag{F.11}$$

G. Details of couplings from $\mathbf{126}+\overline{\mathbf{126}}$-plet mediation

In this appendix we expand the $SO(10)$ coupling structures that enter in $126+\overline{126}$-plet mediation in section 4 and appendicies C and D in a $SU(5)\times U(1)$ basis. We list these structures below

$$f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\Bigg[\frac{4}{\sqrt{5}}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}i}\Bigg] \tag{G.1}$$

$$f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{ij}_{\acute{b}\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\Bigg[\frac{2}{\sqrt{5}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{ij}_{\acute{b}k} + \frac{1}{6}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{a}n}\mathcal{P}^{n}_{\acute{b}klm}$$
$$-\frac{1}{\sqrt{30}}\epsilon^{ijklm}\mathcal{P}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}_{\acute{b}lm}\Bigg] \tag{G.2}$$

$$f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{a}i\mu}\mathbf{P}_{\acute{b}j\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\Bigg[\sqrt{\frac{3}{10}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}_{\acute{b}jk} + \sqrt{\frac{3}{10}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}j}\mathcal{P}_{\acute{b}ik} + \frac{1}{\sqrt{2}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}i}\mathcal{P}^{(S)}_{\acute{b}jk}$$
$$+\frac{1}{\sqrt{2}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}j}\mathcal{P}^{(S)}_{\acute{b}ik}\Bigg] \tag{G.3}$$

$$f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}j\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\left[\frac{\sqrt{6}}{5}\mathcal{P}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}jk} + \sqrt{\frac{2}{5}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{(S)}_{\acute{b}jk} + \mathcal{P}^{\mathbf{T}}_{\acute{a}k}\mathcal{P}^{k}_{\acute{b}j}\right] \tag{G.4}$$

$$\begin{aligned} f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}k\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}} & \left[\frac{1}{2\sqrt{30}}\epsilon^{ijpqr}\mathcal{P}^{s\mathbf{T}}_{\acute{a}pqr}\mathcal{P}_{\acute{b}ks} + \frac{1}{6\sqrt{2}}\epsilon^{ijpqr}\mathcal{P}^{s\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{(S)}_{\acute{b}ks}\right. \\ & -\frac{1}{10}\epsilon^{ijpqr}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}_{\acute{b}kr} - \frac{1}{2\sqrt{15}}\epsilon^{ijpqr}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{(S)}_{\acute{b}kr} \\ & \left. +\mathcal{P}^{ij\mathbf{T}}_{\acute{a}p}\mathcal{P}^{p}_{\acute{b}k} + \frac{1}{2\sqrt{5}}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{i}_{\acute{b}k} - \frac{1}{2\sqrt{5}}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}k}\right] \end{aligned} \tag{G.5}$$

$$\begin{aligned} f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}^{kl}_{\acute{b}\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}} & \left[\frac{1}{6}\epsilon^{ijpqr}\mathcal{P}^{s\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{kl}_{\acute{b}s} + \frac{1}{6}\epsilon^{klpqr}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}s}\mathcal{P}^{s}_{\acute{b}pqr}\right. \\ & +\frac{1}{12\sqrt{5}}\epsilon^{ijpqr}\mathcal{P}^{k\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{l}_{\acute{b}} - \frac{1}{12\sqrt{5}}\epsilon^{klpqr}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}^{j}_{\acute{b}pqr} \\ & -\frac{1}{12\sqrt{5}}\epsilon^{ijpqr}\mathcal{P}^{l\mathbf{T}}_{\acute{a}pqr}\mathcal{P}^{k}_{\acute{b}} + \frac{1}{12\sqrt{5}}\epsilon^{klpqr}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}^{i}_{\acute{b}pqr} \\ & +\frac{1}{10\sqrt{6}}\epsilon^{ijlpq}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{k}_{\acute{b}} - \frac{1}{10\sqrt{6}}\epsilon^{klipq}\mathcal{P}^{j\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}pq} \\ & -\frac{1}{10\sqrt{6}}\epsilon^{ijkpq}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{l}_{\acute{b}} + \frac{1}{10\sqrt{6}}\epsilon^{kljpq}\mathcal{P}^{i\mathbf{T}}_{\acute{a}}\mathcal{P}_{\acute{b}pq} \\ & \left. -\frac{1}{\sqrt{30}}\epsilon^{ijpqr}\mathcal{P}^{\mathbf{T}}_{\acute{a}pq}\mathcal{P}^{kl}_{\acute{b}r} - \frac{1}{\sqrt{30}}\epsilon^{klpqr}\mathcal{P}^{ij\mathbf{T}}_{\acute{a}p}\mathcal{P}_{\acute{b}qr}\right] \end{aligned} \tag{G.6}$$

$$\begin{aligned} f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\epsilon_{jklmn}\mathbf{P}^{kl\mathbf{T}}_{\acute{c}\mu}\mathbf{P}^{mn}_{\acute{d}\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}} & \left[4\mathcal{P}^{pq\mathbf{T}}_{\acute{c}r}\mathcal{P}^{r}_{\acute{d}pqj} - 4\sqrt{\frac{2}{15}}\mathcal{P}^{\mathbf{T}}_{\acute{c}pq}\mathcal{P}^{pq}_{\acute{d}j}\right. \\ & \left. -\frac{4\sqrt{6}}{5}\mathcal{P}^{p\mathbf{T}}_{\acute{c}}\mathcal{P}_{\acute{d}pj}\right] \end{aligned} \tag{G.7}$$

$$\begin{aligned} f^{(\overline{126})(+)}_{\acute{a}\acute{b}}\mathbf{P}^{ij\mathbf{T}}_{\acute{a}\mu}\mathbf{P}_{\acute{b}j\mu} = f^{(\overline{126})(+)}_{\acute{a}\acute{b}} & \left[-\frac{1}{2\sqrt{30}}\epsilon^{iklmn}\mathcal{P}^{p\mathbf{T}}_{\acute{a}lmn}\mathcal{P}_{\acute{b}kp} - \frac{1}{6\sqrt{2}}\epsilon^{iklmn}\mathcal{P}^{p\mathbf{T}}_{\acute{a}lmn}\mathcal{P}^{(S)}_{\acute{b}kp}\right. \\ & \left. +\frac{1}{10}\epsilon^{iklmn}\mathcal{P}^{\mathbf{T}}_{\acute{a}mn}\mathcal{P}_{\acute{b}kl} + \mathcal{P}^{ki\mathbf{T}}_{\acute{a}l}\mathcal{P}^{l}_{\acute{b}k} - \frac{1}{2\sqrt{5}}\mathcal{P}^{k\mathbf{T}}_{\acute{a}}\mathcal{P}^{i}_{\acute{b}k}\right] \end{aligned} \tag{G.8}$$

$$\bar{f}^{(126)(+)}_{\acute{c}\acute{d}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}_{\acute{d}\nu} = \bar{f}^{(126)(+)}_{\acute{c}\acute{d}}\left[\frac{4}{\sqrt{5}}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}}\mathcal{Q}_{\acute{d}k}\right] \tag{G.9}$$

$$\begin{aligned} \bar{f}^{(126)(+)}_{\acute{c}\acute{d}}\mathbf{Q}^{\mathbf{T}}_{\acute{c}\nu}\mathbf{Q}_{\acute{d}ij\nu} = \bar{f}^{(126)(+)}_{\acute{c}\acute{d}} & \left[\frac{2}{\sqrt{5}}\mathcal{Q}^{\mathbf{T}}_{\acute{c}k}\mathcal{Q}^{k}_{\acute{d}ij} + \frac{1}{6}\epsilon_{ijklm}\mathcal{Q}^{n\mathbf{T}}_{\acute{c}}\mathcal{Q}^{klm}_{\acute{d}n}\right. \\ & \left. -\frac{1}{\sqrt{30}}\epsilon_{ijklm}\mathcal{Q}^{k\mathbf{T}}_{\acute{c}}\mathcal{Q}^{lm}_{\acute{d}}\right] \end{aligned} \tag{G.10}$$

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$$
\begin{aligned}
\bar f^{(126)(+)}_{\acute c\acute d}\,\mathbf{Q}^{i\mathbf{T}}_{\acute c\nu}\mathbf{Q}^{j}_{\acute d\nu} = \bar f^{(126)(+)}_{\acute c\acute d}\Bigg[&\sqrt{\frac{3}{10}}\,\mathcal{Q}^{i\mathbf{T}}_{\acute c k}\mathcal{Q}^{jk}_{\acute d} + \sqrt{\frac{3}{10}}\,\mathcal{Q}^{j\mathbf{T}}_{\acute c k}\mathcal{Q}^{ik}_{\acute d} \\
&+\frac{1}{\sqrt2}\mathcal{Q}^{i\mathbf{T}}_{\acute c k}\mathcal{Q}^{jk}_{(S)\acute d} + \frac{1}{\sqrt2}\mathcal{Q}^{j\mathbf{T}}_{\acute c k}\mathcal{Q}^{ik}_{(S)\acute d}\Bigg]
\end{aligned}
\tag{G.11}
$$

$$
\bar f^{(126)(+)}_{\acute c\acute d}\,\mathbf{Q}^{\mathbf{T}}_{\acute c\nu}\mathbf{Q}^{j}_{\acute d\nu} = \bar f^{(126)(+)}_{\acute c\acute d}\left[\frac{\sqrt6}{5}\mathcal{Q}^{\mathbf{T}}_{\acute c k}\mathcal{Q}^{jk}_{\acute d} + \sqrt{\frac{2}{5}}\,\mathcal{Q}^{\mathbf{T}}_{\acute c k}\mathcal{Q}^{jk}_{(S)\acute d} + \mathcal{Q}^{k\mathbf{T}}_{\acute c}\mathcal{Q}^{j}_{\acute d k}\right]
\tag{G.12}
$$

$$
\begin{aligned}
\bar f^{(126)(+)}_{\acute c\acute d}\,\mathbf{Q}^{\mathbf{T}}_{\acute c ij\nu}\mathbf{Q}^{k}_{\acute d\nu} = \bar f^{(126)(+)}_{\acute c\acute d}\Bigg[&\frac{1}{2\sqrt{30}}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute c s}\mathcal{Q}^{ks}_{\acute d} + \frac{1}{6\sqrt2}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute c s}\mathcal{Q}^{ks}_{(S)\acute d} \\
&-\frac{1}{10}\epsilon_{ijpqr}\mathcal{Q}^{pq\mathbf{T}}_{\acute c}\mathcal{Q}^{kr}_{\acute d} - \frac{1}{2\sqrt{15}}\epsilon_{ijpqr}\mathcal{Q}^{pq\mathbf{T}}_{\acute c}\mathcal{Q}^{kr}_{(S)\acute d} \\
&+\mathcal{Q}^{p\mathbf{T}}_{\acute c ij}\mathcal{Q}^{k}_{\acute d p} + \frac{1}{2\sqrt5}\mathcal{Q}^{\mathbf{T}}_{\acute c j}\mathcal{Q}^{k}_{\acute d i} \\
&-\frac{1}{2\sqrt5}\mathcal{Q}^{\mathbf{T}}_{\acute c i}\mathcal{Q}^{k}_{\acute d j}\Bigg]
\end{aligned}
\tag{G.13}
$$

$$
\begin{aligned}
\bar f^{(126)(+)}_{\acute c\acute d}\,\epsilon^{jklmn}\mathbf{Q}^{\mathbf{T}}_{\acute c kl\nu}\mathbf{Q}_{\acute d mn\nu} = \bar f^{(126)(+)}_{\acute c\acute d}\Bigg[&4\mathcal{Q}^{r\mathbf{T}}_{\acute c pq}\mathcal{Q}^{pqj}_{\acute d r} - 4\sqrt{\frac{2}{15}}\,\mathcal{Q}^{pq\mathbf{T}}_{\acute c}\mathcal{Q}^{j}_{\acute d pq} \\
&-\frac{4\sqrt6}{5}\mathcal{Q}^{\mathbf{T}}_{\acute c p}\mathcal{Q}^{pj}_{\acute d}\Bigg]
\end{aligned}
\tag{G.14}
$$

$$
\begin{aligned}
\bar f^{(126)(+)}_{\acute c\acute d}\,\mathbf{Q}^{\mathbf{T}}_{\acute c ij\nu}\mathbf{Q}_{\acute d kl\nu} = \bar f^{(126)(+)}_{\acute c\acute d}\Bigg[&\frac{1}{6}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute c s}\mathcal{Q}^{s}_{\acute d kl} + \frac{1}{6}\epsilon_{klpqr}\mathcal{Q}^{s\mathbf{T}}_{\acute c ij}\mathcal{Q}^{pqr}_{\acute d s} \\
&+\frac{1}{12\sqrt5}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute c k}\mathcal{Q}_{\acute d l} - \frac{1}{12\sqrt5}\epsilon_{klpqr}\mathcal{Q}^{\mathbf{T}}_{\acute c i}\mathcal{Q}^{pqr}_{\acute d j} \\
&-\frac{1}{12\sqrt5}\epsilon_{ijpqr}\mathcal{Q}^{pqr\mathbf{T}}_{\acute c l}\mathcal{Q}_{\acute d k} + \frac{1}{12\sqrt5}\epsilon_{klpqr}\mathcal{Q}^{\mathbf{T}}_{\acute c j}\mathcal{Q}^{pqr}_{\acute d i} \\
&+\frac{1}{10\sqrt6}\epsilon_{ijlpq}\mathcal{Q}^{pq\mathbf{T}}_{\acute c}\mathcal{Q}_{\acute d k} - \frac{1}{10\sqrt6}\epsilon_{klipq}\mathcal{Q}^{\mathbf{T}}_{\acute c j}\mathcal{Q}^{pq}_{\acute d} \\
&-\frac{1}{10\sqrt6}\epsilon_{ijkpq}\mathcal{Q}^{pq\mathbf{T}}_{\acute c}\mathcal{Q}_{\acute d l} + \frac{1}{10\sqrt6}\epsilon_{kljpq}\mathcal{Q}^{\mathbf{T}}_{\acute c i}\mathcal{Q}^{pq}_{\acute d} \\
&-\frac{1}{\sqrt{30}}\epsilon_{ijpqr}\mathcal{Q}^{pq\mathbf{T}}_{\acute c}\mathcal{Q}^{r}_{\acute d kl} - \frac{1}{\sqrt{30}}\epsilon_{klpqr}\mathcal{Q}^{p\mathbf{T}}_{\acute c ij}\mathcal{Q}^{qr}_{\acute d}\Bigg]
\end{aligned}
\tag{G.15}
$$

$$
\begin{aligned}
\bar f^{(126)(+)}_{\acute c\acute d}\,\mathbf{Q}^{\mathbf{T}}_{\acute a ij\mu}\mathbf{Q}^{j}_{\acute b\mu} = \bar f^{(126)(+)}_{\acute c\acute d}\Bigg[&-\frac{1}{2\sqrt{30}}\epsilon_{iklmn}\mathcal{Q}^{lmn\mathbf{T}}_{\acute c p}\mathcal{Q}^{kp}_{\acute d} - \frac{1}{6\sqrt2}\epsilon_{iklmn}\mathcal{Q}^{lmn\mathbf{T}}_{\acute c p}\mathcal{Q}^{kp}_{(S)\acute d} \\
&+\frac{1}{10}\epsilon_{iklmn}\mathcal{Q}^{mn\mathbf{T}}_{\acute c}\mathcal{Q}^{kl}_{\acute d} + \mathcal{Q}^{k\mathbf{T}}_{\acute c li}\mathcal{Q}^{l}_{\acute d k} - \frac{1}{2\sqrt5}\mathcal{Q}^{\mathbf{T}}_{\acute c k}\mathcal{Q}^{k}_{\acute d i}\Bigg]
\end{aligned}
\tag{G.16}
$$

The above concludes our analysis of the interactions of the vector-spinors with Higgs multiplets in tensor representations, self-couplings of the vector-spinors, and of the couplings of the vector-spinors with the matter in the spinor 16-plet representations. These couplings are of considerable value in the analysis of spontaneous breaking of the $SO(10)$gauge symmetry, in the analysis of proton life time, and in the analysis of quark-lepton textures and in the study of neutrino masses.

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